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## 13. ABSTRACT (Maximum 200 words)

The 1992-1993 year in Control Theory focused on robotics, optimal control for nonsmooth geometric data, adaptive control and filtering, discrete event systems, and power systems. It brought together mathematicians, electrical engineers and practitioners, who identified current difficulties and challenges.

The 1993-1994 year in Emerging Applications of Probability brought together computer scientists and probabilists; it also dealt with stochastic models in geosystems, hidden Markov processes in speech and vision, stochastic networks and branching processes.

The 1994-1995 year in Waves and Scattering dealt with random waves, numerical methods including hybrid methods (FFT and wavelets), and inverse problems, such as nondestructive evaluation, acoustics, NMR, Electro-magnetic, and ground penetrating radar.

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APPLICATIONS OF PROBABILITY, WAVES AND SCATTERING

FINAL REPORT

AVNER FRIEDMAN

November 29, 1995

U.S. ARMY RESEARCH OFFICE

ARMY GRANT NUMBER: DA/DAAH04-93-G-0197

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## FINAL REPORT

Title of Project: Control Theory and Its Applications, Applications or Probability, Waves and Scattering

Army Grant Number: DA/DAAH04-93-G-0197

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### Abstract:

The 1992-1993 year in Control Theory focused on robotics, optimal control for nonsmooth geometric data, adaptive control and filtering, discrete event systems, and power systems. It brought together mathematicians, electrical engineers and practitioners, who identified current difficulties and challenges.

The 1993-1994 year in Emerging Applications of Probability brought together computer scientists and probabilists; it also dealt with stochastic models in geosystems, hidden Markov processes in speech and vision, stochastic networks and branching processes.

The 1994-1995 year in Waves and Scattering dealt with random waves, numerical methods including hybrid methods (FFT and wavelets), and inverse problems, such as nondestructive evaluation, acoustics, NMR, Electro-magnetic, and ground penetrating radar.

### 1A. List of IMA Volumes in Mathematics and Its Applications Published by Springer-Verlag New York Resulting from the above grant:

- Volume 65: Mathematical Finance (workshop was held on June 14-18, 1993)  
Editors: M.H.A. Davis, D. Duffie, W.H. Fleming and S.E. Shreve
  - Kerry Back, Continuous trading with asymmetric information and imperfect competition
  - Jakša Cvitanić and Ioannis Karatzas, Contingent claim valuation and hedging with constrained portfolios
  - Jakša Cvitanić and Ioannis Karatzas On portfolio optimization under “drawdown” constraints
  - Mark H.A. Davis and Thaleia Zariphopoulou, American options and transaction fees
  - Wendell H. Fleming, Optimal investment models and risk sensitive stochastic control
  - I. Karatzas and N. Karoui, The optimal stopping problem for a general american put-option
  - Peter Lakner with Marco Frittelli, Arbitrage and free lunch in a general financial market model the fundamental theorem of asset pricing
  - L.C.G. Rogers, Which model for term-structure of interest rates should one use?
  - S. E. Shreve, Liquidity premium for capital asset pricing with transaction costs
- Volume 71: Stochastic Networks (workshop was held on February 28-March 4, 1994)  
Editors: Frank P. Kelly and Ruth Williams
  - Florin Avram, Dimitris Bertsimas, and Michael Ricard, Sequencing problems in open queueing networks; an optimal control approach
  - Maury Bramson, Two badly behaved queueing networks
  - S.A. Berezner, D.M. Rose, and Yu.M. Suhov, Starlike networks with synchronization constraints
  - J.G. Dai, Stability of open multiclass queueing networks via fluid models
  - Paul Dupuis and Richard S. Ellis, Large Deviation Analysis of Queueing Systems

- Guy Fayolle and Jean-Marc Lasgouttes, A State-Dependent Polling Model with Markovian Routing
- A. Ganesh and V. Anantharam, Stationary tail probabilities in exponential server tandems with renewal arrivals
- Peter W. Glynn, Large deviations for the infinite server queue in heavy traffic
- J. Michael Harrison, Balanced fluid models of multiclass queueing networks: A Heavy traffic conjecture
- J. Michael Harrison and Viên Nguyen, Some badly behaved closed queueing networks
- Frank Kelly, Dynamic routing in stochastic networks
- P.R. Kumar, Scheduling queueing networks: stability, performance analysis and design
- Harold J. Kushner, A Control Problem for a New Type of Public Transportation System, via Heavy Traffic Analysis
- C.N. Laws, On trunk reservation in loss networks
- Avi Mandelbaum and Gennady Pats, State-dependent queues: approximations and application
- T.S. Mountford and B. Prabhakar, Convergence of departures from an infinite sequence of queues
- Bhaskar Sengupta and Raymond W. Yeung, Matrix product-form solutions and LCFS queue
- Gideon Weiss, On optimal draining of re-entrant fluid lines
- Ivy Hsu and Jean Walrand, Admission control for ATM networks
- R.J. Williams, Semimartingale reflecting Brownian motions in the orthant
- Walter Willinger, Traffic modeling for high-speed networks: Theory versus practice
- Volume 72: Discrete Probability and Algorithms (workshops were held on September 20–24, 1993) and October 18–22, 1993)
 

Editors: David Aldous, Persi Diaconis, J. Michael Steele and Joel Spencer

  - David Aldous, On simulating a Markov chain stationary distribution when transition probabilities are unknown
  - Noga Alon, A note on network reliability (LaTeX available (Steve))
  - Persi Diaconis and Anil Gangolli, Rectangular arrays with fixed margins
  - Persi Diaconis and Susan Holmes, Three examples of Monte-Carlo Markov chains: at the Interface between statistical computing, computer science, and statistical mechanics
  - James Allen Fill and Robert P. Dobrow, The move-to-front rule for self-organizing lists with Markov dependent requests
  - Anant P. Godbole, Daphne E. Skipper, and Rachel A. Sunley, The asymptotic lower bound on the diagonal ramsey numbers: A closer look
  - Anna R. Karlin and Prabhakar Raghavan, Random walks and undirected graph connectivity: A survey
  - Joel Spencer with Prasad Tetali, Sidon sets with small gaps (LaTeX available (Steve))
  - J. Michael Steele, Variations on the monotone subsequence theme of Erdős and Szekeres
  - Dominic Welsh, Randomised approximation schemes for Tutte-Gröthendieck invariants (LaTeX available (Steve))
  - J.E. Yukich, Quasi-additive Euclidean functionals
- Volume 73: Discrete Event Systems, Manufacturing Systems and Communication Networks (workshop was held on May 10–14, 1993)
 

Editors: P.R. Kumar and P.P. Varaiya

- P.E. Caines and Y.J. Wei, Markovian fragments: Complete subtheories of COCOLOG theories
- Edwin K.P. Chong, On-line optimization of queues using infinitesimal perturbation analysis
- Yu-Chi Ho, A new paradigm for stochastic optimization and parallel simulation
- Junjie Hu and Michael Caramanis, Dynamic Set-up Scheduling of Flexible Manufacturing Systems: Design and Stability of Near Optimal General Round Robin Policies
- R.P. Kurshan, Homomorphic reduction of coordination analysis
- Ranga Mallubhatla, Krishna R. Pattipati, and N. Viswanadham, Discrete-time MARKOV-reward models of production systems
- Jennifer McManis with Pravin Varaiya, Modeling real-time systems Using Rate Automata
- Sanjai Narain and Ritu Chadha, Symbolic discrete-event simulation
- Karen Rudie and Jan C. Willems, Decentralized discrete-event systems and Computational Complexity
- Ali Sharifnia, Starvation-based instability of distributed scheduling policies in non-acyclic fluid and queuing networks
- Volume 74: Adaptive Control, Filtering and Signal Processing (workshop was held on April 12–16, 1993)
 

Editors: K.J. Åström, G.C. Goodwin, and P.R. Kumar

  - Karl J. Åström, Oscillations in systems with relay feedback
  - Er-Wei Bai and Mark S. Andersland, Compatibility of Stochastic and Worst Case System Identification: Least Squares, Maximum Likelihood and General Cases
  - T.E. Duncan, Some results for the adaptive boundary control of stochastic linear distributed parameter systems
  - Babak Hassibi, Ali H. Sayed and Thomas Kailath, LMS is  $H^\infty$  Optimal
  - Ioannis Kanellakopoulos, Adaptive control of nonlinear systems: A tutorial
  - Miroslav Krstić and Petar V. Kokotović, Estimation-based schemes for adaptive nonlinear state-feedback control
  - Harold J. Kushner and Jichuan Yang Stochastic approximation with averaging and feedback: faster convergence
  - P.R. Kumar, An adaptive controller inspired by recent results on learning from experts
  - Lennart Ljung, Building models from frequency domain data
  - A. S. Morse, Supervisory control
  - Karim Nassiri-Toussi and Wei Ren, Potential self-tuning analysis oF stochastic adaptive control
  - B. Pasik-Duncan, Stochastic adaptive control
  - Miloje Radenkovic, Optimality of the adaptive controllers
  - Anders Rantzer, Uncertain real parameters with bounded rate of variation
  - Victor Solo, Averaging methods for the analysis of adaptive algorithms
  - Jing Sun, A multilinear parametrization approach for identification of partially known systems
  - B. Erik Ydstie, with Joy H. Kelly, Design guidelines for adaptive control with application to systems with structural flexibility
  - Anders Rantzer, Uncertainty with bounded rate of variation
  - G. Yin, Adaptive Filtering with Averaging
- Volume 75: Modeling, Mesh Generation, and Adaptive Numerical Methods for Partial Differential Equations (workshop was held on July 6–23, 1993)

Editors: Joseph E. Flaherty, Ivo Babuska, William D. Henshaw, John E. Hopcroft, Joseph E. Oliker, and Tayfun Tezduyar

### Week 1

- Robert E. Barnhill, Gerald Farin, and Bernd Hamann, NURBS and grid generation
- Isabel Beichl and Francis Sullivan, Coping with degeneracies in Delaunay triangulation
- Christoph M. Hoffmann, Geometric approaches to mesh generation
- Robert Schneiders and Jürgen Debye, Refining quadrilateral and brick element meshes
- Mark S. Shephard, Saikat Dey, and Marcel K. Georges, Automatic meshing of curved three-dimensional domains: Curving finite elements and curvature-based mesh control

### Week 2

- Eric Brière de l'Isle and Paul Louis George, Optimization of tetrahedral meshes
- Tao Lin and Hong Wang, A class of error estimators based on interpolating the finite element solutions for reaction-diffusion equations
- Rabi Mohtar and Larry Segerlind, Accuracy-based time step criteria for solving parabolic equations

### Week 3

- Claudio Carlenzoli and Alfio Quarteroni, Adaptive domain decomposition methods for advection-diffusion problems
- Zhangxin Chen,  $L^p$ -posteriori error analysis of mixed methods for linear and quasilinear elliptic problems
- Giovanni M. Cornetti, A characteristic-Galerkin method for the Navier-Stokes equations in thin domains with free boundaries
- Karen D. Devine, Joseph E. Flaherty, Raymond M. Loy, and Stephen R. Wheat, Parallel partitioning strategies for the adaptive solution of conservation law
- Jacob Fish and Vladimir Belsky, Adaptive multi-grid method for a periodic heterogeneous medium in  $1 - D$
- Kamyar Haghghi and Eun Kang, A knowledge-based approach to the adaptive finite element analysis
- Jens Hugger, An asymptotically exact, pointwise, a posteriori error estimator for the finite element method with super convergence properties
- Manfred Koch, A mesh-adaptive collocation technique for the simulation of advection-dominated single- and multiphase transport phenomena in porous media
- J. Tinsley Oden, Weihan Wu, and Mark Ainsworth, Three-step  $H-P$  adaptive strategy for the incompressible Navier-Stokes equations
- J.A. Schmidt, C.R. Johnson, J.C. Eason, and R.S. MacLeod, Applications of automatic mesh generation and adaptive methods in computational medicine
- Barna A. Szabó, Ricardo L. Actis, and Stefan M. Holzer, Solution of elastic-plastic stress analysis problems by the  $p$ -version of the finite element method
- J. Ware and M. Berzins, Adaptive finite volume methods for time-dependent P.D.E.S
- Zhimin Zhang and J.Z. Zhu, Superconvergence of the derivative patch recovery technique and *a posteriori* error estimation

- Volume 76: Random Discrete Structures (workshop was held on November 15–19, 1993)  
Editors: David Aldous and Robin Pemantle
  - David Aldous, Probability distributions on cladograms
  - Robert M. Burton and William G. Faris, Stability of self-organizing processes
  - Amir Dembo and Ofer Zeitouni, Large deviations for random distribution of mass
  - Amir Dembo and Yosef Rinott, Some examples of normal approximations by Stein’s method
  - Luc Devroye and Olivier Kamoun, Random minimax game trees
  - Persi W. Diaconis, Susan Holmes, Svante Janson, Steven P. Lalley, and Robin Pemantle, Metrics on compositions and coincidences among renewal sequences
  - Bert Fristedt, Intersections and limits of regenerative sets
  - Martin Hildebrand, Random Processes of the form  $X_{n+1} = a_n X_n + b_n \pmod{p}$  where  $b_n$  takes on a single value
  - Svante Janson, The second moment method, conditioning and approximation
  - Charles R. Johnson with John H. Drew, The no long odd cycle theorem for completely positive matrices
  - Russell Lyons, How fast and where does a random walker move on a random tree?
  - Sam Northshield, Recurrence, amenability, and the universal cover of graphs
  - Robin Pemantle, Sharpness of second moment criteria for branching and tree-indexed processes. This paper is rejected by Pemantle himself.
  - Robin Pemantle and Yuval Peres, On which graphs are all random walks in random environments transient?
  - Thomas S. Salisbury, Energy, and intersections of Markov chains
  - Paul Erdős, Svante Janson, Tomasz Łuczak and Joel Spencer, A note on triangle-free graphs
- Volume 77: Nonlinear Stochastic PDE’S: Hydrodynamic Limit and Burgers’ Turbulence (workshop was held on March 21–25, 1994)  
Editors: Tadahisa Funaki and Wojbor A. Woyczyński
  - H.P. Breuer and F. Petruccione, Mesoscopic modelling and stochastic simulations of turbulent flows
  - Chih-Chung Chang, Equilibrium fluctuations of nongradient reversible particle systems
  - P.L. Chow, Stationary solutions of two-dimensional Navier-Stokes equations with random perturbation
  - J. Fritz and B. Rüdiger, Approximation of a one-dimensional stochastic PDE by local mean field type lattice systems
  - T. Funaki, K. Uchiyama, and H.T. Yau, Hydrodynamic limit for lattice gas reversible under Bernoulli measures
  - Kenji Handa, On a stochastic PDE related to Burgers’ equation with noise
  - Yiming Hu and W.A. Woyczyński, Shock density in Burgers turbulence
  - Steven Keleti and XB Reed, Jr., Evaluation of spectral behavior for large ensembles of exact solutions to Burgers’ equation for Thomas initial conditions
  - William C. Meecham, Algebraic energy spectra in stochastic problems for the incompressible Navier–Stokes equation; relation to other nonlinear problems
  - S. Molchanov, Reaction-diffusion equations in the random media: localization and intermittency
  - J. Quastel, Diffusion in disordered media
  - A.I. Saichev and W.A. Woyczyński, Model description of passive tracer density fields in the framework of Burgers’ and other related model equations

- S.S. Sritharan, Nonlinear filtering of stochastic Navier-Stokes equation
- Makoto Sugiura, Sharp asymptotics of diffusion processes with small parameter and applications to metastable behavior
- Donatas Surgailis, Intermediate asymptotics of statistical solutions of Burgers' equation
- Ming Zhu, The reversible measures of a conservative system with finite range interactions
- Volume 80: Image Models (and their Speech Model Cousins), (workshop was held on May 2–6, 1994),  
Editors: Stephen E. Levinson and Larry Shepp
  - Charles Byrne, Iterative reconstruction algorithms based on cross-entropy minimization
  - Basilis Gidas and Alejandro Murua, Stop consonants discrimination and clustering using nonlinear transformations and wavelets
  - Gabor T. Herman, Michael Chan, Yair Censor, Emanuel Levitan, Robert M. Lewitt, and T.K. Narayan, Maximum a posteriori image reconstruction from projections
  - Frederick Jelinek, Direct parsing of text
  - Chuanshu Ji, Hierarchical modelling for microstructure of certain brittle materials
  - Joseph A. Kogan, Hidden Markov models estimation via the most informative stopping times for Viterbi algorithm
  - Kevin E. Mark, Michael I. Miller, and Ulf Grenander, Constrained stochastic language models
  - David O. Nelson, Recovering DNA sequences from electrophoresis data
  - Larry Shepp, A summary (exact title to be determined later)
  - Don X. Sun and Li Deng, Non-stationary hidden Markov models for speech recognition
  - Y. Vardi, Applications of the EM algorithm to linear inverse problems with positivity constraints
- Volume 84: Classical and Modern Branching Processes (workshop was held on June 13–17, 1994)  
Editors: Krishna B. Athreya and Peter Jagers
  - K.B. Athreya and A.N. Vidyashankar, Large deviation rates for supercritical and critical branching processes
  - J.D. Biggins, How fast does a general branching random walk spread?
  - B. Chauvin, A. Rouault, Boltzmann-Gibbs weights in the branching random walk
  - Harry Cohn, Stochastic monotonicity and branching processes
  - D.A. Dawson and Y. Wu, Multilevel multitype branching models of an information System
  - F.M. Dekking and E.R. Speer, On the shape of the wavefront of branching random walk
  - Michael Drmota and Vladimir Vatutin, Limit distributions in branching processes with two types of particles
  - J. Geiger and G. Kersting, Depth-first search of random trees, and Poisson point processes
  - Peter Jagers, Towards dependence in general branching processes
  - F.I. Karpelevich and Y.M. Suhov, A criterion of boundedness of discrete branching random walk
  - Marek Kimmel, Quasistationarity in a branching model of division-within-division
  - Population and density dependent branching processes F.C. Klebaner
  - F. Koukiou, Directed polymers in random media and spin glass models on trees
  - Thomas G. Kurtz, Russell Lyons, Robin Pemantle, and Yuval Peres, A conceptual proof of the Kesten-Stigum theorem for multi-type branching processes

- Quansheng Liu and Alain Rouault, On two measures defined on the boundary of a branching tree
- J. Alfredo López-Mimbela and Anton Wakolbinger, Which critically branching populations persist?
- Russell Lyons, A simple path to Biggins' Martingale convergence for branching random walk
- Russell Lyons, Robin Pemantle, and Yuval Peres, Unsolved problems concerning random walks on trees
- Peter Olofsson, Branching processes with local dependencies
- Robin Pemantle, Sharpness of second moment criteria for branching and tree-indexed processes
- Anthony G. Pakes, On the recognition and structure of probability generating functions
- Ibrahim Rahimov, Record values of a family of branching processes
- Serik Sagitov, Limit skeleton for critical crump-mode-Jagers branching process
- Edward C. Waymire and Stanley C. Williams, Markov cascades
- George P. Yanev and Nickolay M. Yanev, Limit theorems for branching processes with random migration stopped at zero

1B. List of Other Papers Received for Future IMA Volumes which are Funded by DA/DAAH04-93-C-0197

- Mathematical Population Genetics (workshop was held on January 24-28, 1994)  
Editors: Simon Tavaré and Peter J. Donnelly
  - Andrew Clark, Theory and applications of rapid-PCR: mispriming
  - D.A. Dawson, Hierarchical and Mean-field Stepping Stone Models
  - S.N. Ethier, On the normal-selection model
  - G. Brian Golding, The effect of selection on genealogies
  - Robert C. Griffiths and Paul Marjoram, An ancestral recombination graph
  - Robert C. Griffiths and Simon Tavaré, Computational methods for the coalescent
  - Rosalind M. Harding, Lines of descent from mitochondrial Eve - an evolutionary look at coalescence
  - Hilde M. Herbots, The structured coalescent
  - Stephen M. Krone, Claudia Neuhauser, and Hyunchung Kang, A note on the stepping stone model with extinction and recolonization
  - Neil O'Connell, Branching and inference in population genetics
  - Norman Kaplan and B.S. Weir, The use of linkage disequilibrium for estimating the recombination fraction between a marker and a disease gene
  - Mary K. Kuhner, Jon Yamato, and Joseph Felsenstein, Applications of Metropolis-Hastings genealogy sampling recombination fraction between a marker and a disease
  - Human demography and the time since mitochondrial eve Paul Marjoram and Peter Donnelly
  - Alan R. Rogers, Population Structure and Modern Human Origins
  - Stanley A. Sawyer, Estimating Selection and Mutation Rates from a Random Field Model for Polymorphic Sites
  - Stephen W. Schaeffer, Molecular population genetics of a phenotypically monomorphic protein in *Drosophila*
  - Mark Stoneking, Recent African origin of human mitochondrial DNA: Review of the evidence and current status of the hypothesis

- Ziad Taib, Branching processes and evolution
- Fumio Tajima, Estimation of the amount of DNA polymorphism and statistical tests of the neutral mutation hypothesis based on DNA polymorphism
- R.H. Ward, Phylogeography of human mtDNA: An Amerindian perspective
- Gunter Weiss, Andreas Henking and Arndt von Haeseler, Distribution of pairwise distances in growing populations
- Stochastic Models in Geosystems, (workshop was held on May 16–20, 1994)  
Editors: Stanislav A. Molchanov and Wojbor A. Woyczyński
  - Keiiti Aki, Seismic coda waves: A stochastic process in earth's lithosphere
  - R.F. Anderson and S.A. Molchanov One Dimensional Random Walk in a Random Media
  - Cascade of scaling gyroscopes: lie structure, universal multifractals and self-organized criticality in turbulence Y. Chigirinskaya, D. Schertzer
  - L. Ju. Fradkin, A non-linear model for fluid parcel motions in the presence of many large and meso-scale vortices
  - Roman E. Glazman, Scale-dependent ocean wave turbulence
  - Vijay K. Gupta and Ed Waymire A survey of cascades with applications from geosciences
  - J.R. Herring, The role of statistical models in turbulence theory
  - Greg Holloway, Ocean circulation: Flow in probability under statistical dynamical forcing
  - V. Klyatskin and D. Gurarie, Random topography in geophysical models
  - V.I. Klyatskin and W.A. Woyczyński, Dynamical and statistical characteristics of geophysical fields and waves and related boundary-value problems
  - W. Kohler, G. Papanicolaou and B. White, Localization of low frequency elastic waves
  - Peter Müller, Stochastic forcing of oceanic motions
  - Catherine Naud, D. Schertzer, and Shaun Lovejoy Radiative transfer in multifractal atmospheres: fractional integration, multifractal phase transitions and inverse problems
  - S. Pecknold, S. Lovejoy, and D. Schertzer The morphology and texture of anisotropic multifractals using generalized scale invariance
  - L. Piterbarg, Short-correlation approximation in models of turbulent diffusion
  - Murray Rosenblatt, Comments on estimation and prediction for autoregressive and moving average nonGaussian sequences
  - A.I. Saichev and W.A. Woyczyński, Probability distributions of passive tracers in randomly moving media
  - Shandarin, Three-dimensional Burgers' equation as a model for the large-scale structure formation in the universe
  - Hubert Shen, Non-mean field approach to self-organization of landforms via stochastic merger
  - Asymptotics of solutions of Burgers' equation with random piecewise constant data Donatas Surgailis
  - P.L. Taylor and B. Lin, Modeling the spatiotemporal dynamics of earthquakes with a conservative random potential and a viscous force
  - Craig L. Zirbel and Erhan Çinlar, Mass transport by Brownian flows
- Computational Wave Propagation (September 19–23, 1994)  
Editors: Bjorn Engquist and Gregory A. Kriegsmann
  - William L. Kath, Phase-sensitive amplification of pulses in nonlinear optical fibers
  - Brian J. McCartin Control region approximation for electromagnetic scattering computations

- Thomas Hagstrom, On high-order radiation boundary conditions
- T. Hagstrom and S.I. Hariharan, Progressive wave expansions and open boundary problems
- Thorkild B. Hansen, Formulation of spherical near-field scanning in the time domain
- Heinz-Otto Kreiss, Numerical solution of problems with different time scales II
- G.A. Kriegsmann, Microwave heating of materials
- Michael J. Miksis and Lu Ting, Structural Acoustic Interactions and On Surface Conditions
- Andrew N. Norris and Thorkild B. Hansen, Wavefield representation using compact and directionally localized sources
- Modeling sound propagation in the ocean Michael B. Porter
- Wavelets, Multigrid and other Fast Algorithms (Multiple, FFT) and Their Use in Wave Propagation (October 17–21, 1994)
 

Editors: Gregory Beylkin, Ingrid Daubechies, and George Papanicolaou

  - W.C. Chew, J.M. Song, C.C. Lu, R. Wagner, J.H. Lin, H. Gan, and M. Nasir, Fast algorithms for solving electromagnetic scattering problems
  - Stefan A. Sauter, The panel clustering method in 3-d BEM
- Waves in Random and other Complex Media (November 14–18, 1994)
 

Editors: Robert Burridge, George Papanicolaou and Leonid Pastur

  - The conjugate operator method: application to Dirac operators and to stratified media Anne Boutet de Monvel and Radu Purice
  - A. Figotin and P. Kuchment, 2D photonic crystals with cubic structure: asymptotic analysis
  - Jean-Pierre Fouque and Josselin Garnier, On waves in random media in the diffusion-approximation regime
  - V. Freilikher, M. Kaveh, M. Pustilnik, I. Yurkevich, J. Sanches-Gil, A. Maradudin, and Jun Q. Lu Coherent Effects in Scattering from, bounded random systems with discrete spectrum
  - Kenneth M. Golden, The interaction of microwaves with sea ice
  - Sergey Gredeskul, Masha Zusman, Yshai Avishai and Mark Azbel, Electron in two-dimensional system with point scatterers and magnetic field
  - Matti Lassas, Inverse spectral problems for random bodies
  - R.C. McPhedran, N. A. Nicorovici, L. C. Botten and Bao Ke-Da Green's function, lattice sums and Rayleigh's identity for a dynamic scattering problem
  - Haruo Sato, Study of Seismogram Envelopes Based on the Energy Transport Theory
  - M.M. Sigalas, C.-T. Chan and C.M. Soukoulis, Propagation of electromagnetic waves in two-dimensional disordered systems
  - Bart A. Van Tiggelen and Roger Maynard, Reciprocity and coherent backscattering of light
  - P. K. A. Wai and C. R. Menyuk, Physical models of polarization mode dispersion
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  - A proof of the strong Scott conjecture—talk at IMA on May 24, 1995 [Draft] Heinz Siedentop
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### 3. Research Reports by Long Term Visitors: 1992-1993 Control Theory and its Applications

**Hector J. Sussmann** prepared the following assessment of the program: I spent the 1992-93 academic year at the IMA., participating in the organization and the activities of the Control Theory program. My own research benefitted tremendously from the possibility of interacting with various long-term and short-term visitors. Naturally, when I look at the research that I did at the IMA. and the one I am doing

now, it is very hard for me to draw a clear line between the work that I probably would have done anyhow because it was a direct continuation of my previous work, and the work that owes its existence to the IMA year. There are, however, a few examples where the decisive role of the IMA year is particularly clear, so I shall start by describing these.

The workshop on Nonsmooth Analysis and Differential Geometric Methods in Optimal Control, held in February, 1993, brought together a number of researchers representing both approaches. Until this workshop, there had been little contact between these two subcultures with Optimal Control Theory. During the workshop, many of us were able to engage in extensive discussions with people representing the "other side," and this has led to the birth of new directions of research where both approaches are combined. I myself am now actively pursuing one of these directions. Specifically, the visit of Prof. Martino Bardi, from Italy, who gave a couple of lectures on viscosity solutions of first order partial differential equations and the viscosity solution approach to the problem of the characterization of the Value Function in deterministic optimal control, made me renew my interest in this issue, on which I had worked about four years earlier, and about which I had taught a course at Rutgers. Prof. Bardi's lectures included a list of several important open problems in the theory, such as the question of the characterization of the value function as the unique positive viscosity solution of the Bellman equation for linear quadratic optimal control. It turned out that the techniques I had developed in my Rutgers course made it possible to solve this and other problems. So after extensive discussions with Bardi, I contacted a Rutgers student who had the notes of the course and had them typed at the IMA., and I am now working on a number of papers, some on my own, and some with Bardi, based on these notes. This could only have happened in a setting such as that of the IMA, where one had plenty of time for discussion after the regular lectures. In the specific case of my conversations with Bardi, it took us several days until the precise correspondence between my techniques and his formulation of the problems and techniques became clear.

Another important example of work that I am currently doing that owes in its existence to the IMA year is a paper I am writing on a new version of the Pontryagin Maximum Principle under weaker hypotheses than all previous versions (including the Nonsmooth Analysis version of "the Maximum Principle under Minimal Hypotheses" due to F. Clarke) and with stronger conclusions (including high-order necessary conditions for optimality). I had been interested in the Maximum Principle for a long time, and had used various control problems, such as local controllability or the optimal control of large mechanical systems that occur in Robotics, and for these problems there was never any difficulty arising from lack of smoothness of the data. I had thought quite a lot about nonsmooth versions of the Maximum Principle, but I had never been sufficiently motivated to pursue this activity. The IMA year, and in particular the February workshop, provided the motivation. Several experts on the Nonsmooth approach — in particular, R.T. Rockafellar and B. Borkukhovitch—discussed versions of the Nonsmooth Maximum Principle, and this made me aware of two things: (a) that there was a great lack of awareness among the differential-geometrically oriented practitioners of control theory, such as myself, of the importance of extending our results to Nonsmooth settings, and (b) that among the nonsmooth analysts there existed a misperception that geometric methods were of more restricted applicability because they required more smoothness. Based on my own thoughts on the subject, I became convinced that it had to be possible to extend the very best results of geometric optimal control—which included things such as high-order conditions that could not be handled by Nonsmooth methods—to general situations where smoothness assumptions were not made. This eventually became a true theorem whose proof I found in August of 1993. I am now completing a paper where a very general version of the Maximum Principle under minimal conditions is proved. Although the actual writing of this work is taking place after the end of the IMA control year, the work owes its existence to the IMA year, in particular to the discussion with visitors that took place during the February workshop.

During the visit of Matthias Kawski, from Arizona State University, we spend a long time discussing ideas about a reformulation of the theory of the Chen series and of iterated integrals based on combining some earlier work of mine on the "product expansion of the Chen series" with some new algebraic techniques developed by Kawski. This had led to a collaboration now in progress, whose final output is a paper that we are now writing, in which we give a proof of the product expansion for a much more general class

of basis of a free Lie algebra than those considered in the original version, where only P. Hall bases were treated.

My former student Wensheng Liu spent the year at the IMA as a postdoctoral fellow. In collaboration with him we obtained some rather significant new results on abnormal subriemannian minimizers. Until October 1992, only two examples of abnormal minimizers—one due to R. Montgomery, the other one to I. Kupka—were known, and in both cases the optimality proof was extremely complicated and appeared to depend on very special features of the examples that could not be generalized. With Liu we found a method that gives a simple proof of optimality and applies to large classes of trajectories. It has now become possible to prove that huge families of abnormal extremals, including all generic ones for two-dimensional distributions, are locally optimal. This work probably fits the category of “resultings generated by the IMA year” slightly less well than the others, in the sense that it took place during the IMA year but was not the result of a short-term visit or a workshop. However, it certainly would not have happened if it had not been for the fact that Liu and I were there together and the opportunity to spend long hours discussing this problem.

The above discussion of the effect of the IMA control year on my own work provides just one illustration of how much was achieved during the year. Other visitors will tell about their own experiences, and I probably should not speak for them, but from my own conversations with several of these visitors I know that in many cases significant new results were obtained and new directions of research were born. For example, Jan Willems, from Groningen, in the Netherlands, spent three months at the IMA, where he met Karen Rudie, who was there for the whole year as a postdoctoral fellow. Although their areas of interest were in principle quite different, it turned out that the set of concepts that Rudie had been developing in order to formulate a general definition of “discrete event system” was closely related to the ideas of Willems on dynamical systems, and this led to a collaboration in which a new version of Willems’ general theory, incorporating discrete events, was developed.

Besides its direct effect on the research of participants such as myself, the control theory year contributed in an important way to our professional activity by helping us widen our knowledge and become acquainted with the current state of the arts in fields of control theory other than our own special research area. This is particularly significant for an area such as control theory, which is mathematically very diverse, so that usually a large investment of time and effort is required—even for an experienced member of the control community—to learn about development in neighboring fields. Among the many events that enabled us to acquire a new perspective of other fields within control theory, I would particularly single out the workshop on Fuzzy Logic and its Applications. As is well known, the evaluation of the applications of Fuzzy Logic—a large number of which are currently being carried out in Japan—is a hotly contested subject, on which opinions have been expressed ranging all the way from extremely enthusiastic to highly critical. The format chosen for the Fuzzy Logic workshop, in which scientists directly involved in specific applications presented their work, and extensive discussions followed, made it possible for most of us to become much better informed about the existing applications and about the controversies surrounding them.

Summarizing, I personally regard the IMA control year as having had a very significant impact on my own research, and I think that its effect on the whole field of control theory as been felt by most specialists in the area. The 1992-93 IMA year will be remembered for a long time as a major event in our field.

### Bingyu Zhang

During the 1992-1993 program year at the IMA I have mainly conducted the following two research projects:

#### *Stability and Robustness Analysis of Control Laws for Nonlinear Aerospace Systems:*

This is a two year joint project with Blaise Morton of *System Research Center of Honeywell Inc.*. In the past we have been concentrated on stability and robustness analysis of *dynamic inversion* control law for nonlinear aerospace systems.

In practical applications, engineers use simple linear models of their systems and linear systems theory to design feedback control laws. These linear control laws are then tested extensively by numerical simulation on a nonlinear "truth model" before implementation in the real system. The linear algorithms usually work surprisingly well, considering the crudeness of the models used. It is an interesting problem in dynamical systems to analyze the nonlinear models directly and prove the desired systems properties hold. The practical value of such an analysis is that the simulations are expensive and do not necessarily reveal all potential flaws in the design.

Dynamic inversion is a nonlinear control technique that has been applied by *Honeywell* to a variety of realistic aerospace vehicle models with reasonable good results. The list of study applications includes models of the F-14 aircraft, the HARV F-18 aircraft, a McDonnell Douglas model of the NASP vehicle, and a General Dynamics model of a next-generation booster vehicle. The main advantage of dynamic inversion over more conventional linear control techniques is its applicability to the full nonlinear vehicle models.

Stability and robustness analysis are very important for further applications of dynamic inversion control law for nonlinear aerospace system. In our recent paper [1], we are able to provide a global stability result with a rigorous mathematical proof for some nonlinear aircraft pitch-axis models. Application of this result to F-14 leads us to obtain a stability criterion based on which one can see easily whether the system is globally stable or not by only checking the engine thrust and the elevator angle.

In addition, the stability result obtained in this paper is robust with respect to the uncertain lift and drag aerodynamic coefficients. The robustness analysis of the stability result with respect to the pitch moment is a more challenging problem which we plan to address in our next research.

*Papers:*

1. *Stability of dynamic inversion control laws applied to nonlinear aircraft pitch-axis models* (with Blaise Morton and Dale Enns), preprint.
2. *Taylor series expansion for solutions of the Korteweg-de Vries equation with respect to their initial values*, IMA preprint series #1015.
3. *Analyticity of solutions of the Generalized Korteweg-de Vries equation with respect to their initial values*, IMA preprint series #1040.
4. *A remark on the Cauchy problem for the Korteweg-de Vries equation on a periodic domain*, IMA preprint series #1155.
5. *Smoothing and decay properties of solutions of the Korteweg-de Vries equation on a periodic domain with point dissipation* (with David Russell), IMA preprint series #1083.
6. *On initial value problem for the forced Korteweg-de Vries equation* (with Jerry Bona), in preparation

### Bo Egardt

I have had the privilege to participate in the IMA program on Control Theory and its Applications during the period April 1–April 30, 1993. The special events during this period were a Tutorial on Adaptive Systems, a Workshop on Adaptive Control, Filtering and Signal Processing, and a Minisymposium on Fuzzy Control.

My background is a PhD in adaptive control and an 8-year period in industry, giving experience of process control in general and adaptive control in particular. A few years ago, I returned to the academic field and became professor of control. The stay at IMA has given me a most valuable chance to focus entirely on research for a limited period of time. It has also given me the opportunity to reestablish a number of personal contacts and to establish new ones, something which is sometimes difficult in an industrial capacity.

The focus of my work during the IMA period has been the adaptive field, partly because this is the area of my particular expertise, and partly because this was the area of concentration at IMA during my stay. The issues of particular interest during the IMA stay were:

- Stability of adaptive control systems. This is an area of long-lasting interest. Talks at the workshop given by L. Praly and E. Ydstie, in particular, triggered interesting discussions on both fundamental questions on the necessity of certain algorithmic modifications to achieve stability, and also on proof techniques. Concerning the latter, it was interesting for me to observe that strong links exist between current investigations and my own PhD thesis that dates more than 10 years back.
- Frequency domain characterization of adaptive controllers. I have recently spent some time to investigate how available results concerning frequency domain properties of identification algorithms can be used to help giving design guidelines for adaptive controllers. I have a talk at the workshop on the subject. I noted that there are interesting connections with both the coupling between identification and control as discussed by K.J. Åström and the stability issues mentioned above. The work is continued in cooperation with a PhD student of mine.
- Fuzzy control. The minisymposium on Fuzzy Control gave me an opportunity to get more familiar with this concept, and I also participated in the panel discussion on the subject. Based on a very pragmatic view of fuzzy control as a particular way of doing interpolation, G. Goodwin from Newcastle, Australia and myself initiated an investigation of how far this can be exploited. We are interested in both descriptive and implementation issues. This work was initiated at IMA, but since we both stayed for a short period, the work has to be carried on in remote cooperation between the two of us.

**Maurizio Falcone** spent 2 months in residence and reports as follows:

During my visit to the IMA I continued my researches on the numerical approximation of the value function and of optimal trajectories/controls for optimal control problems in finite dimension. In particular, I have studied problems with state constraints considering two different models. In the first model, we assume that the controlled system of differential equations  $R^n$

$$(1) \quad \dot{y}(s) = g(y(s), \alpha(s)) \quad s \geq 0$$

$$(2) \quad y(0) = x$$

describing the dynamics satisfies a boundary condition, namely that at each point of the boundary of the constraint  $\Omega$  there exists a control such that the corresponding vectorfield is pointing inward. In the second model, we do not make assumptions on the vectorfield at the boundary  $\partial\Omega$  but we assume that is possible to modify (1) adding a term so that the resulting trajectory satisfies the constraint (this corresponds to a projection on the constraint set). By dynamic programming one can prove that the value function  $u$  satisfies a Hamilton–Jacobi–Bellman equation. Moreover, using the techniques related to viscosity solutions, one can prove that  $u$  is the unique viscosity solution of that equation satisfying generalized Dirichlet boundary conditions (in the first model) or generalized Neumann boundary conditions (in the second model).

For the first problem, I have obtained *a priori* estimates in  $L^\infty$  for the convergence of an approximation scheme based on a discretization in time and space of the original control problem. This discretization leads to an approximate Hamilton–Jacobi–Bellman equation which can be actually solved by a fixed point algorithm. These results will appear in the proceedings of the workshop “Nonsmooth Analysis and Geometric Methods in Deterministic Optimal Control.”

For the second problem, I have obtained (in collaboration with J.L. Menaldi, Wayne State Univ.) an existence and uniqueness result for the Cauchy problem related to the “adaptive” dynamics. Moreover, we have used this result to show that the value function is the unique viscosity solution of a Hamilton–Jacobi–Bellman equation with Neumann boundary conditions and we have developed an approximation scheme for the value function. These results will appear in a preprint of the IMA.

I should also mention that this visit gave me the opportunity to establish scientific contacts with many other researchers coming from different places. This has been a very useful in particular to start new projects. For example, after attending the workshops “Control and Optimal Design of Distributed

Parameter Systems" and "Flow Control" I started working on a new research program related to the numerical approximation of control problems in infinite dimension. I also had the opportunity to present the results of my researches in the Control Seminar, in the workshop on "Nonsmooth Analysis" and at the Department of Mathematics of Wayne State University which I visited for one week.

**W.H. Fleming** spent 2 months in residence as an Ordway Visiting Professor of the School of Mathematics. He reports on his current research on Risk Sensitive Stochastic Control.

There are various approaches to treating disturbances in control systems. In Stochastic control, disturbances are modelled as stochastic processes (random noise.) On the other hand, in robust control theory disturbances are modelled deterministically. The theory of risk sensitive optimal control provides a link between stochastic and deterministic approaches.

For linear systems with quadratic cost criteria,  $H$ -infinity – optimization provides a method for robust control design. The disturbance attenuation problem is one of those considered in robust  $H$ -infinity – control theory. If a state space formulation is used, an associated "soft constrained" differential game arises naturally. The stochastic control counter part is a linear exponential quadratic regulator (LEQR) problem, introduced by Jacobson.

An interesting question is to find for nonlinear systems, or nonquadratic cost criteria, similar connections between stochastic and robust control approaches to disturbance attenuation problems. Whittle introduced an interesting approach to this question using large deviations ideas. Whittle considered problems on a finite time horizon, and used Freidlin-Wentzell type "small noise" asymptotics. Methods of viscosity solutions for nonlinear partial differential equations are useful in proving asymptotic results of this kind.

In current research, stochastic control problems on an infinite time horizon with exponential cost criteria are considered. The Donsker-Varadhan large deviation rate is used as a criterion to be optimized. The optimum rate is characterized as the value of an associated stochastic differential game, with an ergodic (expected average cost per unit time) cost criterion. By taking a small-noise limit a deterministic differential game with average cost per unit time cost criterion is obtained. This differential game is related to robust control of nonlinear systems.

*Papers in preparation*

1. W.H. Fleming and W.M. McEneaney, Risk sensitive control on an infinite time horizon
2. W.H. Fleming (for proceedings of IMA Workshop June 14–18, 1993 on Mathematical Finance.

**Pramod Khargonekar** reports:

I was a long term visitor during September–December 1992 in the Special Year on Control on Theory at the Institute for Mathematics and its Applications.

I gave 3 lectures on the basic multivariable control systems in the tutorial workshop during the week of September 7th. The basic idea was to give a general background on control theory to the post-docs and other participants.

Along with Bruce Francis, we had a very successful workshop on Robust Control Theory during the week of September 21, 1992. We had lectures from some of the brightest younger researchers in the field. The lectures were held in the morning and early afternoon. Each day was devoted to a specific sub-area within the robust control field. During later afternoon, we had an open discussion session which was coordinated by a moderator. These discussion sessions were highly stimulating and were a great success. We had many people who said that they enjoyed these sessions as they were informal while at the same time thought provoking. On the whole, I felt that this workshop was very successful. A book based on the workshop is under preparation at the IMA.

I gave an hour-talk in the workshop on advanced control systems organized by Professors Enns, Morari, and Nett. My topic was, at the special request of the organizers was on the newly emerging area of control of semiconductor manufacturing equipment. Our work was very preliminary at the time of the talk and it has since matured significantly. As a matter of fact, I will give a plenary talk on this topic at the upcoming

IEEE Conference on Decision and Control in December 1993. This talk will be an updated version of the talk I gave at the IMA.

I gave a talk in the Control Science seminar at the invitation of Dr. Balas of the Aerospace Department.

A major achievement during my stay at the IMA was that I renewed my collaboration with Professor Allen Tannenbaum. Allen and I had done a lot of joint work at Florida and Minnesota. However, after my move to Michigan, there was no opportunity to continue this collaboration. We began to work together again during my stay at the IMA. A paper based on some of the early results was presented at the Conference on Information Systems and Sciences at the Johns Hopkins University. A full paper based on that work in collaboration with H. Bercovici and C. Foias has been submitted for journal publication. Both Allen and I plan to continue this new collaboration that began during my stay at the IMA.

In addition, I continued to do the research that I was doing at Michigan.

In summary, I had a very fruitful stay at the IMA. The environment was very stimulating. I had a lot of fun talking to the various participants.

**P.R. Kumar** was a program coordinator and spent about 5 months at the IMA. He reports on the following activities:

- (i) I organized the tutorial week on Adaptive Control, Filtering and Signal Processing, April 5-9, 1993. This was a unique forum featuring the leading experts in the world, including Professors Karl Åström and Graham Goodwin, and was an in depth presentation covering both the theoretical as well as practical and industrial aspects of the field.
- (ii) I organized the Workshop on Adaptive Control, Filtering and Signal Processing, April 12-16, 1993. This was a first rate theoretical workshop covering both linear and nonlinear systems, discrete and continuous time, stochastic and deterministic systems.
- (iii) I organized the tutorial week on Discrete Event Systems, Manufacturing Systems and communication Networks, May 3-7, 1993. Unfortunately, one of the speakers had to cancel at the last moment. The topics of discrete event systems and manufacturing systems were covered in three hours each.
- (iv) I organized the Workshop on Discrete Event Systems, Manufacturing Systems and Communication Networks, May 10-14, 1993. This was a forum featuring experts in all three fields, and provided opportunity for interdisciplinary contact.
- (v) I organized the Mini-symposium on Fuzzy Control, April 19-20, 1993. This featured a carefully selected set of speakers from Japan, who were involved with many of the pioneering applications. In this country, and in fact at many countries except Japan, the views of this field are formed from second hand sources. Here we had a unique opportunity to calibrate the field. Especially interesting were the talks by Dr. Seiji Yasunobu, a first rate designer of several of the fuzzy control systems (e.g. The Sendai train system, the fuzzy crane, and the road tunnel ventilation system). These gave us an invaluable insight into the domains where fuzzy control is successful in real world applications. All who attended the tutorials that I talked to felt they now understood the heart of the field. My own bottom line is this: Fuzzy control has been useful in situations where the dynamics is not fast, and when it is used in a predictive control mode. In spite of much reading in the field, I, and others that were present, had not been provided with that insight into the field.
- (vi) I presented two seminars on manufacturing systems, one at IMA, and the other in the Department of Electrical Engineering during the semester.
- (vii) I presented a tutorial on learning theory in the tutorial week on Adaptive Control.
- (viii) I presented a lecture in the Workshop on Adaptive Control.
- (ix) I presented a three hour tutorial on manufacturing systems. This is a very "hot" area, and I am planning on writing a book on the topic. I was able to use part of my stay to organize my thoughts in this field.

- (x) I presented a lecture in the Workshop on Discrete Event Systems.
- (xi) I am working on two manuscripts, one on a learning theory based adaptive controller, and the other on queuing networks. Part of the research was done while at the IMA.

**Suzanne Lenhart** reports

I really enjoyed my 3-month visit to the IMA this fall. I appreciated the opportunity to participate in the IMA activities.

During my visit to the IMA, I wrote a paper.

“Bilinear Optimal Control of a Kirchhoff Plate”, jointly with M.E. Bradley. We have submitted the paper to Systems and Control Letters. A bilinear control is used as a force to make the plate close to a desired profile taking into account, a quadratic cost of control. We prove the existence of an optimal control and characterize it uniquely through the solution of an optimality system.

I also wrote a paper,

“Optimal Control for Degenerate Parabolic Equations with Logistic Growth,” with Jiongmin Yong. This paper has been accepted for publication in Nonlinear Analysis Theory, Methods, and Applications Journal. The paper considers optimal control of a degenerate parabolic partial differential equation governing a diffusive population with logistic growth terms. Assuming the population (like beavers) causes damage to forest and agricultural land, the optimal control is the trapping rate and the cost functional is a combination of the damage and trapping costs.

I also finished a paper,

“Optimal Control of a Convective-Diffusive Fluid Problem,” which will be submitted to Quarterly of Applied Mathematics. The paper treats optimal control of a parabolic equation, modeling one-dimensional fluid flow through a soil-packed tube in which a contaminant is initially distributed. A fluid is pumped through the tube to remove the contaminant. The convective velocity due to the fluid pumping is the control. The goal is to minimize a performance criterion which is a combination of the total contaminant at the final time and the cost of the control.

I enjoyed my interactions with Bradley, Yong, and many other IMA visitors. The living arrangements were satisfactory and I had suitable daycare for my son.

**Thomas A. Posbergh** was in residence for most of the program year. He reports:

In recent years powerful methods have emerged from the applied mathematics community which have furthered our understanding of nonlinear systems. Research into the stability of geometrically exact structures in the context of hamiltonian systems with symmetry has led to a new techniques known as the energy-momentum method which exploits the underlying mechanical structure. Application of this method leads to a set of inequalities whose satisfaction ensures the nonlinear stability of the system of interest. More importantly however, in the case of infinite dimensional systems, the method identifies the set of coordinates in which the flexible dynamics of the system can be decoupled from the rigid body modes with a significant simplification of the problem.

Application of the energy-momentum method in the area of nonlinear control has lead to a new results. Our recent work has explored the application of these techniques to the design of robust feedback control for uniformly rotating mechanical systems. In Zhao & control gains which stabilize uniform rotation about an arbitrary axis in a rigid body. Through a set of inequalities characterizing stable motion this method explicitly identifies the effect of particular model parameters in ensuring stability of rotation. Given a feedback gains which will ensure robust stability of uniform rigid body rotation. To our knowledge this result, described in detail in the cited papers, is one of the few robust control law designs for a nonlinear system. The result has a satisfying physical interpretation and can be made sharp.

This work is documented in the following papers which were completed or written during the IMA year in control. Both papers benefited greatly from discussions with other visitors to the IMA during that time.

R. Zhao and T.A. Posbergh [1993a] "Feedback Stabilization of Uniform Rigid Body Rotation," to appear in *System and Control Letters*, 12 pages.

R. Zhao and T.A. Posbergh, "Robust Stabilization of a Uniformly Rotating Rigid Body," submitted to *IEEE Transactions on Automatic Control*, 41 pages, 1993.

At present we have preliminary results extending this methodology to the model of a flexible structure modeled as a geometrically exact rod. We are currently documenting these results. This preliminary work has yielded stabilizing control laws for the axial rotation of a uniformly rotating flexible rod. As part of this effort we are interested in numerical simulation and graphical display of the dynamics to further our understanding and complement the analytical effort. In work at the Army High Performance Computer Research Center at the University of Minnesota a numerical simulation based on an energy and momentum preserving algorithm is in the process of being coded and debugged. This second order accurate algorithm exhibits exact conservation of both energy and momentum. The simulation is coded as a straight forward finite element implementation and accurately models the long term nonlinear dynamics of the flexible rod.

### Laurent Praly

The main topic of my research is the design of stabilizing controllers. Precisely, for a controlled finite dimensional dynamical system :

$$\frac{dx}{dt} = f(x, u), \quad x \in IR^n, \quad u \in IR^m,$$

the problem is :

given a function  $h$ , find an integer  $k$  and two functions  $\varphi_1$  and  $\varphi_2$  such that the solutions of :

$$\begin{cases} \frac{dx}{dt} = f(x, u) \\ \frac{d\xi}{dt} = \varphi_1(\xi, h(x)), \quad \xi \in IR^k \\ u = \varphi_2(\xi, h(x)) \end{cases}$$

are well defined, remain in a prescribed set and their  $x$ -components tend to a desired rest point.

My stay for more than two months at the IMA has been an exceptional opportunity for me to make what I consider as important progress on various specifications of this problem. This has been made possible by the means offered by the IMA, its very efficient organization and the so many opportunities of top class and very fruitful exchanges of ideas these two allow. To be very precise I shall mention three collaborative projects for which my stay at the IMA was of prime importance :

1. With A. Teel, one of the organizers of the IMA period of concentration on : Nonlinear feedback design, we have considered the problem of the minimal information about the pair  $(f, h)$  needed to get a solution. The results we have obtained are the object of a paper we are finishing :

*L. Praly, A. Teel*

*Sufficient prior information for global output practical regulation by output feedback*

*To be submitted for publication in SIAM J. Control and Optimization.*

2. With W. P. Dayawansa, visiting the IMA at the beginning of March 93, we have finished a paper where we exhibit pairs of functions  $(f, h)$  as well as we formalize the corresponding obstruction such that the problem has no solution.

*F. Mazenc, L. Praly, and W. P. Dayawansa*

*Global stabilization by output feedback : Examples and Counter-Examples.*

*Submitted for publication in Systems & Control Letters. April 1993.*

3. With E. Ydstie, visiting the IMA in April 93, our stay has been for us the first opportunity of having a long period where we could concentrate fully on our project of a book on adaptive linear control. Adaptive linear control concerns the case where the functions  $f$  and  $h$  are linear but unknown. Our topic is to present the main tools and the results on the boundedness and the properties of the solutions. The progress which has been made possible is such that we expect to be done by the end of the summer.

These three projects are only examples. I have attended four workshops. I had many discussions with people whose only concern was to present and explain fully their latest results and to exchange freely ideas, problems and techniques. The consequence is that I learned so much that I expect my stay will pay off for several years. One of the expected results concerns the case of systems which generalizes the following form :

$$\left\{ \begin{array}{l} \frac{dx_1}{dt} = x_2 + f_2(x_2, x_3, \dots, x_n, u) \\ \frac{dx_2}{dt} = x_3 + f_3(x_3, \dots, x_n, u) \\ \vdots \\ \frac{dx_{n-1}}{dt} = x_n + f_n(x_n, u) \\ \frac{dx_n}{dt} = u + f_{n+1}(x_1, x_2, \dots, x_n, u) \end{array} \right.$$

I have been learning a lot on such systems from the discussions I had at the IMA with A. Teel, H. Sussmann and E. Sontag. From this an important research project has grown up.

**Anders Rantzer** reports:

As a long term visitor from September -92 to June -93, I was able to follow the entire IMA program in control. The opportunities for interaction with other researchers were outstanding and very rewarding.

My work during the year was devoted to problems arising in robustness analysis of linear finite-dimensional systems. Four major projects can be distinguished.

1. Robustness of a dynamical system can be defined in different ways, depending on the purpose of the analysis. Usually, some uncertain element is introduced to perturb the system equations and one studies how the behavior of the model is affected by the perturbation. Structural assumptions about the uncertain element are essential. For example, a bounded nonlinear or time-varying operator can obviously distort variables much more than a scalar multiplication satisfying the same bound.

Recent research in the area shows for several different types of uncertainty, that necessary and sufficient conditions for stability can be written on the form of a convex optimization problem. The type of uncertainty only affects the domain of optimization.

During the year at IMA, I spent much time together with John C. Doyle, also a long term visitor, trying to systematize these ideas. Currently, material is being collected for a survey paper.

2. In the work described above, it was noted that there is a large gap between results available for uncertain constant parameters on one hand and on the other hand uncertain parameters that vary arbitrarily with time. This is disturbing and it would often be desirable to exploit bounds on the time-variations. In particular, this is true in many applications where slowly time-varying systems are analyzed using time-invariant models.

With this motivation and inspired by recent work of A. Megretskii (IMA visitor in October -93) I tried to exploit bounds on the frequency content of uncertain parameters. The work resulted in a new criterion of the form described in 1. and a short publication [4] was prepared. Jan Willems, another fall time IMA visitor, kindly pointed out close connections to his own work from the sixties on periodic perturbations.

3. From November on, I was sharing office at IMA with Bo Bernhardsson and Li Qiu, who both work in my area of research. Hence, we decided to look for some good common project. It turned out that Li and his former adviser Ted Davison had a nice conjecture about a linear algebra problem that arises very naturally in control theory. The problem states as follows: *Given a stable real matrix  $A$  (all eigenvalues in the open left half plane), what is the smallest (induced) norm of a real matrix  $\Delta$  such that  $A + \Delta$  is unstable.*

After a few weeks of work and some suggestions from Peter Young and John Doyle, we were able to solve the problem (just in time to have it printed in the IFAC Congress Proceedings [2]). It also turned out that the solution has relations to several apparently different problems in linear algebra and complex analysis.

4. Finally, I had the opportunity to continue some joint work with A. Megretskii [5, 1, 3] on synthesis of linear controllers that maximize stability robustness with respect parametric uncertainty. Our main achievement is to show that the maximization can be stated in terms a linear functional with affine matrix inequality constraints. This makes the problem tractable for computations.

#### References

- [1] A. Megretskii and A. Rantzer. Robust control synthesis by convex optimization: Projective parametrization and duality. In *Proceedings of IFAC Congress, Sydney, Australia*, 1993.
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**Jan C. Willems** spent fall quarter 1992 at the IMA. He reports

#### CONTROL IN A BEHAVIORAL CONTEXT

I worked in two different areas:

1. Control theory from a behavioral point of view.
2. Control of discrete event systems.

The second area constitutes on-going joint work with Dr. Karen Rudie.

I would like to take this opportunity to thank you, Hector Sussmann, Avner Freedman, and all of the IMA staff for having supported my visit. It was a very fruitful time for me.

As for publications originated at the IMA are concerned please note the following two

– F. Fagnani and J.C. Willems. Symmetries of Differential Systems to appear CONTROL THEORY, DYNAMICAL SYSTEMS AND GEOMETRY OF DYNAMICS (Eds. K.D. Elworthy, W.N. Everitt, and E.B. Lee) Festschrift at the occasion of the 70-th birthday of L. Markus Marcel Dekker, Inc.

– J.C. Willems LQ-control: A Behavioral Approach Submitted for presentation at the 1993 IEEE Conference of Decision and Control

The behavioral approach to dynamical systems starts by taking the time-trajectories which a dynamical system produces as its starting point, without departing, as is common, from its input/output structure. In this project, this approach was extended to control problems. Let  $\mathcal{L}^q$  denote the set of linear time-invariant differential systems in  $q$  variables. Thus each element of  $\mathcal{L}^q$  is defined by a system of differential equations

$$R \left( \frac{d}{dt} \right) w = 0$$

where  $R(\xi)$  is a polynomial matrix with  $q$  columns in the indeterminate  $\xi$ . Let  $\mathcal{B}$  be the behavior of this system, i.e., the set of solutions of this system of differential equations.

Let  $L \in IR^{q \times q}[\xi, \eta]$  be a two-variable polynomial matrix. Associate with  $L$  the quadratic differential form

$$q_L(w) = \sum_{m,n} \left( \frac{d^m w}{dt^m} \right)^T L_{m,n} \left( \frac{d^n w}{dt^n} \right)$$

Now consider the following optimal control problem defined by the system  $R$  and the quadratic form  $L$ . Call a trajectory  $w^* \in \mathcal{B}$  optimal if  $\lim_{t \rightarrow \infty} w^*(t) = 0$  and if for all  $\Delta \in \mathcal{B}$  of compact support there holds that the cost-degradation

$$\int_{-\infty}^{+\infty} (q_L(w^* + \Delta) - q_L(w^*)) dt$$

is non-negative.

The main result obtained is a characterization of  $\mathcal{B}^*$ , the set of *optimal* trajectories. It has been shown that  $\mathcal{B}^*$  is non-empty if and only if

$$L(-i\omega, i\omega) \geq 0$$

for all  $\omega \in IR$ . If this is the case, then  $\mathcal{B}^*$  consists of the trajectories of

$$L \left( -\frac{d}{dt}, \frac{d}{dt} \right) w = 0$$

which approach zero as  $t \rightarrow \infty$ .

Other results include a further characterization of  $\mathcal{B}^*$  in terms of quadratic polynomial matrix equations reminiscent of the Riccati equation and of spectral factorizations.

In addition to these concrete research results, I initiated a number of new research directions while I was at the IMA. In particular, I worked in collaboration with dr. Karen Rudie, an IMA post-doc, on the computational complexity of decentralized discrete event control problems. Together with Prof. John Doyle, another IMA visitor, I discussed formulations for modelling system uncertainty in the framework of the behavioral approach to dynamical systems. I expect that this will lead to a very fruitful research direction for me in the years to come.

**E. Zuazua** spent fall quarter 1992 at the IMA. He reports:

This quarter at the IMA was devoted to Control of PDE. We have continued our research in that field as well as in other topics of PDE. This is a brief abstract of the results we have obtained. For clarity we distinguish three different research subjects:

1. *Control of elastic systems.*

In a joint work with S. Hansen (IMA) we have studied the controllability and stabilization of a string with point-masses. This is a model example of "hybrid-system". These systems have attracted a lot of attention recently.

We solve both the control and the stabilization problems by using methods of characteristics and non-harmonic Fourier series.

The most interesting result is probably the following one: One may control a string with interior point masses from a free-end. However the space of initial configurations one may control decreases by one order of derivability each time we cross a mass. This phenomena is related to the well-posedness of this system when the regularity one requires to the left and to the right of the point mass is not the same. This fact was unknown. We have proved it analyzing the propagation of singularities across the mass.

We understand this work will lead to an intensive study of the controllability and stabilizability of multi-dimensional flexible systems with masses.

2. *Control of heat processes.*

With C. Fabre and J.P. Puel we have written a series of three works on the controllability of semilinear heat equations.

Due to the strong smoothing effect of the heat equation one can not expect exact controllability results but only approximate ones. But the notion of approximate controllability is difficult to deal with in nonlinear systems.

In this article we have developed a new technique that allows us to solve several approximate controllability problems (boundary-control, interior-control, control at the initial value) when the nonlinearities are globally Lipschitz. We also show how one may construct bang-bang controls that minimize the  $L^\infty$ -norm among all the admissible ones.

There is still a lot to be done in the nonlinear frame but we understand this (first) step is important.

#### 4. Research Reports by Long Term Visitors: Emerging Applications of Probability

Annual Program Organizers:

**J. Michael Steele** of the University of Pennsylvania chaired the Organizing Committee for the year in “Emerging Applications of Probability” and was in residence at the IMA for nine months during the 1993–94. Here are his comments on the 1993–94 year at the IMA:

##### *Retrospective on the Special Year in Applied Probability.*

The bottom line is that one could not have imagined the year to have been more successful. Some of the results first promulgated during the special year seem destined to become the focus of many years of future research and admiration. The two such results that I have in mind are Talagrand’s Isoperimetric Inequality and Yuval Perez’ new Capacity Theorem (and its applications to points of multiplicity).

From my perspective, these results are extraordinary, but the success of the year should be measured more broadly. All of the workshops really did “work.” I had a special appreciation of the Fall workshops, because of having had a good hand in their organization, and again because of this I found them quite inspiring. Not too long ago there was no serious probabilistic theory of trees, but now the subject—led substantially by Aldous—is undeniably rich. Similarly with the new theory of finite Markov chains, where one now systematically exploits connections to differential geometry and PDE. Even two years ago, almost one could have imagined these connections or their effectiveness on concrete, discrete problems arising in computer science.

The workshops of the Spring term are farther from my expertise, but I have no trouble seeing that the workshop of Peter Donnelly and Simon Tavaré was an exceptional success. The witches’ brew that made this workshop so visibly effective was the blend of theoretical geneticist and of individuals with real field experience. The same combination was also afoot in the workshop on “Hidden Markov Models and their Speech Cousins” that was organized by Steve Levinson and Larry Shepp. There were more core participants of the mathematical genetics workshop who were in residence at the IMA for a good stretch, and this, I think, added a lot to the overall effectiveness of the genetics workshop.

#### David Aldous

This is a report on the projects I worked on during my stay at the IMA, Fall quarter 1993. Projects 1 – 3 were small projects which were essentially completed during my stay; projects 4 – 6 are larger, long-term projects on which progress was made.

1. By simulating a Markov chain for sufficiently many steps one can simulate a pick from its stationary distribution. Paper [2] gives a rigorous method of determining how many steps are enough, using only the simulation and not *a priori* assumptions on the chain. (After presenting this work at a Workshop, Peter Winkler said that he and Lovasz had been casually thinking about a related problem, so perhaps my work will encourage them to work through the details of their approach.)
2. One of the mathematically most interesting parts of mathematical population genetics is the circle of ideas surrounding the Ewens sampling formula and probability distributions on partitions. In [3] I study whether an analogous set of ideas works for the study of random phylogenetic trees, representing simple models for relationships between species.
3. Stochastic models for changes in population size with time are a classical topic in applied probability. Applying such models to the origin and extinction of species is possible but seems conceptually less satisfying. As an alternative, [1] describes simulations of a rule-based model. This involves mathematical representations of notions such as genetic type of species, environmental niche, fitness

- of a species in a niche, and adaptation. There are underlying random mechanisms for changes of niche sizes and for disconnection and reconnection of geographical regions, and these ultimately drive the evolution of species.
4. There is a broad circle of known theory relating to the length  $L_n$  of the longest increasing subsequence of a random  $n$ -permutation. In almost-finished work joint with Persi Diaconis we show that a continuous limit process, which we call *Hammersley's process*, exists as a continuous-space interacting particle process. This turns out to be analogous to the (discrete-space) simple asymmetric exclusion process, and also to be an elaboration of Hammersley's Poisson process representation. The celebrated result  $EL_n \sim 2n^{1/2}$  is intimately tied to the hydrodynamical limit theorem for Hammersley's process.
  5. The topic of the October workshop was "Finite Markov Chain Renaissance". I am in the middle of writing a book on this topic, intended as a systematic account of theory intermediate between introductory Markov chain theory and current research. This project goes back 5 years, when I first taught a course on the topic, and there has already been quite a lot of interaction between the course notes and the subsequent development of the topic.
  6. At the IMA I started on what I expect to be my main research project over the next year. This involves theoretical study of Metropolis-type optimization algorithms in the setting of tree-indexed objective functions. Ongoing work of Pemantle, Lyons and Peres presented at the November workshop is closely related to this project, so perhaps some collaboration will emerge in future.

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### *Miscellaneous comments.*

First let me say that I enjoyed my stay at the IMA very much, and appreciate all the work done by you and your staff. I must confess that I used my time mostly to complete projects which I hadn't previously had time to think carefully about, rather than to embark upon new projects, so I don't have anything to put in the "new collaborative projects" category. In the finite Markov chains world, I am often used as a repository of known results and methods, so I like to think I help the field along even though I'm not writing joint papers. For example, Presad Tetali (who was at the October workshop) had a conjecture, and I pointed out this followed from some 10-year old work in a different area. As to "evaluation of the entire program", I think it was completely successful, and I heard only positive comments during my stay. We did our job of bringing together people in slightly different departmental categories – probability, computer science, discrete math – who worked on similar problems but might not usually see one another, and the sensible ones took advantage of the opportunity to learn something.

### **Persi Diaconis**

In my very active time at IMA, I interacted with many departments. I gave the Math and Statistics Colloquia, spoke in the Combinatorics Seminar and gave a lecture in Peter Webbs Group Theory Course. I also gave a university wide lecture "On Coincidences."

I also ran a seminar on Orthogonal Polynomials and their use in Probability and Combinatorics. This was done jointly with Dennis Stanton and involved speakers from the IMA (Miller, Belsley, Janson, Diaconis) and elsewhere. This led to joint work with Stanton which is ongoing.

One of the joys of visiting was working with Hans Weinberger who helped me understand how to use continuous techniques to bound eigenvalues of graphs. Again, we are working on joint projects. I also benefited from interacting with young group theorists; this has certainly affected my current work (I'm giving a lecture on a theorem I learned today!) Further, I had a healthy interaction with Charles Zeyer and Joe Eaton in Statistics; this has led to papers on all sides.

Two examples of interactions withing the IMA: Svante Janson, Steve Lahey, Susan Holmes solved some problems in renewal theory that arose in applied statistical work. The paper will appear in the IMA Proceedings a second success, I managed to interest Steve Lalley in the Ergodic Theoretic aspects of my work on Shuffling Cards. He has created a lovely new theory which allows us to solve a host of problems previously deemed intractable.

Finally, I had many former graduate students there, Hildebrand, Belsley, Pemantle, Mathews; also Jim Fill who works closely with me. I think I tricked each of them into doing something new. Further David Aldous and I finished up our joint work on Longest Increasing Sequences.

#### Workshop Organizers:

**J. Michael Steele** of the University of Pennsylvania organized the September 9-15, 1993 tutorials on *Probability and Optimization* and was in residence at the IMA for nine months during 1993-94. (See also his comments above). Here is his report on the tutorials:

The purpose of this set of lectures by Michael Steele, Persi Diaconis and Joel Spencer was to provide an introduction to some of the basic research issues that would be addressed in the first two workshops of the Special Year in the Emerging Applications of Probability.

The first group of topics considered were those surrounding the theory of Subadditive Euclidean Functionals. This theory provides a reasonably unified way of studying the behavior of many of the classical problems of combinatorial optimization theory in the context of natural probability models. The leading example of such a problem is the Euclidean Travelling Salesman Problem (TSP) which concerns the length of the shortest path through  $n$  points in the plane.

This problem has evoked voluminous research over the years because it is computationally difficult, yet it can be approached by a variety of heuristic methods. It is also of importance in the context of the simplest stochastic model where the points are chosen at random in the unit square. The early work of Beardwood, Halton, and Hammersley on the behavior of the length of the TSP path has inspired much probabilistic work—several aspects of which were reported upon in this tutorial as well as in subsequent workshops. Salient among the later contributions were those of J. Yukich on the “rooted dual” method, and especially the contribution of M. Talagrand on his subadditive Euclidean functionals of several types, but the method also provides vital information in several problems of concern in the classical theory of sums of vector-valued random variables.

In his contributions to the tutorial, Persi Diaconis reported on the remarkable progress that has been made in the area of rates of convergence to stationarity of finite Markov chains. The rapid development of this topic is due to several sources, but the algorithmic applications originated by Jerrum and Sinclair provide the central reason that this area has experienced a recent renaissance.

The contribution of Joel Spencer to the tutorial was an elegant introduction to the more recent and most refined uses of the famous “probabilistic method” that was introduced into combinatorics by P. Erdős in the pre-dawn days of applied probability. The lecture by Spencer gave insight into the use of Lovasz’ local lemma and other probabilistic tools that serve as fundamental methods for the uses of probability in combinatorics.

#### Joel Spencer

##### Talagrand’s Inequality

This is really a nice story that shows what a positive effect IMA can have. Let  $\Omega = \prod \Omega_i$  be a product probability space,  $A \subset \Omega$ . Talagrand defines, for  $t > 0$ , a “fattening”  $A_t$  containing  $A$ . To get a rough idea when  $\Omega = \{0, 1\}^n$  is the Hamming cube then  $y \in A_t$  implies  $y$  is within Hamming distance  $t\sqrt{n}$  of  $A$ . Talagrand proves  $\Pr[A] \Pr[\overline{A}_t] < e^{-t^2/4}$ . Roughly, if  $A$  is moderate and  $t$  is large then  $A_t$  has most all the space.

Mike Steele showed this to a whole group of us. He showed (as Talagrand knew) how to use this to give sharp concentration results for certain random variables. We (Eli Shamir, Svante Janson, Dominic Welsh, myself, and others) started working on it and we (esp. Svante) came up with a general application. Let  $h : \Omega \rightarrow \mathbb{R}$  be a random variable. Suppose  $h(x)$  is not too strongly affected by changing one coordinate of  $x = (x_1, \dots, x_n)$ . Suppose further that if  $h(x) \geq a$  then there is some “small” set of the coordinates  $x_i$  that “certify” that  $h(x) \geq a$ . Then one gets a strong concentration of  $h$  around its mean. Talagrand himself then came to a workshop and we had further discussions. We’ve looked at concentration for a number of classical problems. Consider the job assignment problem where the  $i$ -th person in the  $j$ -th job gains a random  $a_{ij}$  and  $X$  is the gain with the optimal assignment. Then  $X$  is strongly concentrated. Consider Minimal Spanning Tree with distances  $\rho(x, y)$  being random and  $Y$  the size of the MST. Then  $Y$  is strongly concentrated. (Janson is working on much more precise results in this case.) I have an IMA Jan ’94 preprint that uses these ideas to give an alternate (and I think cleaner) proof of the Erdős-Hanani conjecture (first shown by Rödl) on asymptotic packings. I received a preprint by Alan Frieze and Bruce Reed on clique coverings and using these ideas could greatly simplify the proof. It seems clear to me (nothing written yet) that studies of extension statements for random graphs (e.g., every vertex lies in a triangle) will be greatly aided by this idea. Overall, it is a very exciting new concept.

What role did IMA play? The original proof was due to one man (Talagrand) thinking by himself. But the understanding of what the result meant and the dissemination to the general community (which is now fully underway) might not have taken place without the fortuitous grouping of us at IMA and the time available there to explore the potentials.

#### *A problem of Erdős*

When Uncle Paul comes to town there are always good problems. Paul asked about the largest induced bipartite graph in a trianglefree graph with  $n$  vertices and  $e$  edges. With Janson and Luczak we were able pretty much to solve this and it will appear in the Proceedings of the Random Structures workshop.

#### *Large Deviations*

I benefitted tremendously by numerous discussions with Svante Janson on large deviations. For example, let  $X$  be the number of empty bins when  $m$  balls are thrown randomly into  $n$  bins. We saw how to estimate  $E[e^{tX}]$  and how to use this to get good bounds on the probability of  $X$  being far from its mean. I don’t think a specific paper will come of this but it was a case of ideas being clarified and distilled.

#### **Robin Pemantle**

During my stay at the IMA I worked (as I usually do) on a variety of different projects. During the first couple of months, I worked mostly on continuations of projects I brought with me, in particular, (2), (5) and (6) below. The IMA is a good place to talk to people and I eventually began working on some other projects, which are not listed below because it’s not clear they amount to anything yet. One is work with M. Eaton on the relationship between admissibility and recurrence of a certain random walk. Another is a project with Yossi Rinott (who visited for 2 weeks) on negatively dependent random variables. The collaboration (3) below, came directly from “hallway” conversations. During the November workshop, R. Lyons and Y. Peres were at the IMA, which allowed us to continue our collaboration (4). And the survey paper, (1), I had the luxury of working on in the background when other things were slow. Oh yes, and the IMA Blues was co-authored with L. Cowen, I. Hueter, S. Petrone and J. Rosenthal. Here follows a description of the results from (4), obtained between October and December 1993. Consider a Galton-Watson branching process with finite mean: each individual independently has a random number of children, the numbers being IID with probability  $p_k$  of  $k$  children. A single progenitor generates a family tree which may be finite or infinite. This tree has a size-biased version, which may be described by a simple recursive algorithm. Size-biasing removes one moment, thus magnifying the tail of the distribution. If the original distribution fails to have  $k p_k \log k$  summable, then the new distribution  $q_k$  fails to have  $q_k \log k$  summable, which means its tails are fat enough that the size-based tree grows superexponentially. This magnification allows the elementary proof of some classical and difficult analytic theorems on branching processes. Some of this, we knew before. But just now, we discovered that you can un-size-bias, to recover the original tree, conditioned to survive at least  $n$  generations. If you keep track of the descendants

affected by un-size-biasing, you find elementary proofs of the exponential and gamma limit laws as well – elementary in that no generating functions or calculations are necessary.

Articles written in large part at the IMA:

- (1) Tree-Indexed Processes.
- (2) Sharpness of second moment criteria for branching and tree-indexed processes.
- (3) Coincidences among renewal sequences.  
(Joint with P. Diaconis, S. Holmes, S. Janson and S. Lalley.)
- (4) Conceptual proofs of  $L \log L$  criteria for mean behavior of branching processes.  
(Joint with R. Lyons and Y. Peres.)

Other articles I revised or added to during my stay at the IMA:

- (5) A shuffle that mixes sets of any fixed size much faster than it mixes the whole deck.
- (6) Martin capacity for Markov chains and random walks in varying dimensions.  
(Joint with I. Benjamini and Y. Peres.)

#### 4. Research Reports by Long Term Visitors: 1993–1994 Emerging Applications of Probability

##### Simon Tavaré

My principle research focus during my month at the IMA was statistical inference in population genetics, in particular the development of computer-intensive methods of inference for complex stochastic processes.

Population geneticists have studied the distribution of allele frequencies in populations for a long time. With the advent of rapid DNA sequencing techniques and the polymerase chain reaction, there are now many studies of molecular variability at the population level. This variability is represented by a set of DNA sequences coming from a sample of individuals. A comparison of the sequences leads to inferences about population structure and history, and estimates of parameters such as mutation, migration, recombination, and selection rates.

These estimates are based on the probability distribution (under some stochastic model for sequence evolution) of summary statistics derived from the DNA sequences, such as the number of types observed in the sample. Unless simple summary measures are used, these sampling distributions are notoriously intractable. In joint work with R. Griffiths (Monash University), I have been developing Markov chain Monte Carlo methods for computing sampling distributions. The idea is as follows: we use the probabilistic structure of the model to derive a recursion satisfied by the sampling probabilities. The coefficients in the recursion are then used to construct a Markov chain in such a way that the required sampling probability  $q$  is the mean of a functional of the chain. Repeated simulation of independent trajectories of the chain then provides an estimate of  $q$ . Importance sampling can be used to generate a Monte Carlo approximant to the likelihood surface  $q_\theta, \theta \in \Theta$ , where  $\Theta$  is the parameter space of interest.

This Markov chain Monte Carlo technique is remarkable generic, in the sense that many different problems can be attacked with it. This has led to joint projects with R. Ward (Utah), S. Schaeffer (Penn State), and B. Golding (McMaster) that are exploring applications to human mitochondrial data and *Drosophila* sequences. Papers written with the support of IMA are:

Unrooted genealogical tree probabilities in the infinitely-many-sites model. *Math. Biosci.*, 1994, in press. With R.C. Griffiths.

Ancestral inference in population genetics. *Statistical Science*, 1994, in press. With R.C. Griffiths.

find a control  $f$  so that the system  $(*) - (**)$  has a solution  $u(x, t)$  satisfying

$$u(x, 0) = \phi(x), \quad u(x, T) = \psi(x).$$

If the control  $f$  is allowed only to act on a small subset of the domain  $(0, 2\pi)$ , then the same result still holds if the initial-terminal states,  $\psi$  and  $\phi$ , have small “amplitude” in certain sense.

In the case of *closed loop* control, the distributed control  $f$  is assumed to be generated by a linear feedback law conserving the “volume” while monotonically reducing  $\int_0^{2\pi} u(x, t)^2 dx$ . The solution of the resulting closed loop system are shown to be exponentially uniformly decay to a constant state. As in the open loop control case, small amplitude assumption is needed if the control is allowed only to act on a small subdomain.

In the project with Jerry L. Bona, we considered the forced KdV equation. The initial-value problem for the KdV equation with a forcing term has recently gained prominence as a model for a number of interesting physical situations. At the same time, the modern theory for the initial-value problem for the unforced KdV equation has taken great strides forward. The mathematical theory pertaining to the forced equation has not kept up with recent advances. In our project, this aspect is rectified with the development of a broader theory for the initial-value problem for the forced Korteweg-de Vries equation. The results obtained include analytic dependence of solutions on both the initial condition and the forcing and allows the external forcing to lie in function classes sufficiently large that a Dirac  $\delta$ -function or its derivative is included. **Papers**

1. Stability of dynamic inversion control laws applied to nonlinear aircraft pitch-axis models (with Blaise Morton and Dale Enns), *IMA Preprint Series*, # 1245, July 1994.
2. Robustness analysis of dynamic inversion control laws applied to nonlinear aircraft pitch-axis models (with Blaise Morton), preprint.
3. Exact controllability and stabilizability of the Korteweg-de Vries equation (with David L. Russell), *IMA Preprint Series*, #1244, July 1994.
4. Boundary stabilization of the Korteweg-de Vries equation, preprint.
5. On initial value problem for the forced Korteweg-de Vries equation (with Jerry Bona), preprint.

### Ruth J. Williams

Below is the report of my activities at the IMA. I have included a report on

- (1) My own research,
- (2) Interactions of others,
- (3) the Stochastic Networks workshop

### INTERACTIONS WITH IMA VISITORS

During her three month stay at the IMA, Professor Williams interacted with visitors in the three areas of diffusion approximations to multiclass queueing networks, passage time moments for reflected diffusions, and probabilistic methods for solving nonlinear elliptic equations. These interactions are described in more detail below.

## (a) Diffusion approximations to multiclass queueing networks.

One of the most intriguing open problems for multiclass queueing networks is to determine conditions for the stability of these networks and for approximating them in heavy traffic by reflected diffusion processes. A related problem is that of determining conditions for positive recurrence of the reflected diffusion processes. Paul Dupuis and Ruth Williams have shown that stability of certain dynamical systems implies positive recurrence for these reflected diffusion processes. Motivated by this result, Jim Dai has shown that stability of certain fluid models implies positive recurrence of related queueing networks. At the IMA, Professor Williams had discussions with Jim Dai and Gideon Weiss concerning their recent work on using piecewise linear Lyapunov functions to determine sufficient conditions for stability of fluid models. She also discussed the use of Lyapunov functions to determine conditions for positive recurrence of reflected random walks with Michael Menshikov and Vadim Malyshev. Professor Williams is now investigating whether piecewise linear Lyapunov functions might be used to show stability of the dynamical systems of Dupuis and Williams mentioned above. This is part of a larger project to determine concrete conditions for stability of reflected diffusion processes that arise as approximations to multiclass queueing networks.

## (b) Passage time moments for reflected diffusion processes.

At the IMA Professor Williams learned of the work of Michael Menshikov and his coauthors on conditions for finiteness of passage time moments of one-dimensional discrete time semimartingales. These authors applied their results, together with a martingale functional of Varadhan and Williams, to determine conditions for the finiteness of passage time moments of reflected random walks in a quadrant. It is natural to try to extend these semimartingale results to continuous time and to apply them to reflected Brownian motions in a quadrant. Professor Williams is collaborating with Michael Menshikov on such a project. To date some progress has been made, but some difficulties still remain in generalizing the discrete time results to continuous time. Assuming this can be done, it is intended to seek other applications of the semimartingale results, for example to other diffusion processes besides reflected Brownian motions in a quadrant.

## (c) Probabilistic methods for solving nonlinear elliptic partial differential equations.

Professor Williams is pursuing a mixed probabilistic and analytic approach to solving nonlinear elliptic equations with various boundary conditions in non-smooth domains. Some results that have already been obtained with Z. Q. Chen and Z. Zhao for semilinear elliptic equations were presented at the IMA during the period of concentration on probabilistic methods for solving nonlinear partial differential equations. Chen, Williams and Zhao are currently trying to extend these results to include equations with a nonlinear first order derivative term. One of the key ingredients needed for this is a Green function estimate for a linear operator with possibly singular first order and zero order coefficients, in a Lipschitz domain. Discussions, initiated at the IMA, are continuing with Professor Eugene Fabes on how to derive such an estimate.

#### INTERACTIONS OF OTHER VISITORS

Jim Dai and Bruce Hajek were good catalysts for discussions on stochastic networks during the three month Winter period. Informal discussions played at least as important a role as formal talks. To give but one example, during a dinner conversation, Bruce Hajek mentioned an open problem for ring networks and the next day Jim Dai presented a solution based on a technique he was developing for determining sufficient conditions for stability of multiclass networks. The four weeks surrounding the stochastic networks workshop were especially lively with the presence of a solid core of experts. Michael Harrison and Frank Kelly were especially instrumental in creating a friendly atmosphere in which they shared their expertise with other participants. The area of stochastic networks is in rapid development. Although I do not know that major open problems were resolved during the IMA period devoted to this subject, I do know that there was progress on understanding their complexity and subtlety and I expect that we will see some major breakthroughs in this area in the next few years, fueled by the interactions and connections that were made at the IMA.

## STOCHASTIC NETWORKS WORKSHOP

The Stochastic Networks workshop held February 28-March 4, 1994, and organized by Frank Kelly and Ruth Williams, was the core for the period of activity on this topic. Stochastic networks are currently used in studying telecommunications, computer networks, and manufacturing systems. The workshop featured presentations on a variety of open problems for stochastic networks. A notable feature of the workshop was the way in which experts in different areas such as operations research, systems science, and mathematics, converged to discuss problems of common interest from different viewpoints. It is planned that the workshop proceedings volume will convey the current state of knowledge in this rapidly developing area.

**Wojbor A. Woyczyński** of Case Western Reserve University was an organizer for the workshops on *Stochastic Models in Geosystems* (May 16-20, jointly with Stanislav Molchanov) and on *Stochastic Methods for Nonlinear PDE's* (March 21-25, jointly with T. Funaki). He participated in IMA activity for five months of 1994. He is currently in the process of editing two IMA volumes containing proceedings of the two workshops that he organized. The first is entitled "Nonlinear Stochastic PDE's: Hydrodynamic Limit and Burgers' turbulence" (jointly with Funaki) and is in the final stages of preparation with 15 papers already deposited for final formatting at IMA and two in final stages of preparation by the authors. The second, entitled "Stochastic Models in Geosystems" (coedited with Molchanov), is about 70 percent complete. It will contain contributions from about 18 authors.

His report follows:

### *Workshop on STOCHASTIC MODELS IN GEOSYSTEMS:*

Thirty three talks were presented by researchers from US, France, Canada, Great Britain, Russia, Italy, Ukraine, China and Lithuania, but many other scientists participated. Besides mathematics, they represented several disciplines including statistics, geology, oceanography, atmospheric sciences, physics and astrophysics. The major topics were turbulent diffusion and passive tracer transport in random velocity flows with applications to oceanic and atmospheric transport, stochastic forcing of oceanic motions, stochastic processes in Earth's lithosphere, propagation of waves in random media and multifractal stochastic space-time models. The speakers provided an excellent overview of current research and several open problems were suggested. New contacts were established.

### *Concentration period on STOCHASTIC METHODS IN NONLINEAR PDE'S:*

This was a more focused meeting concentrating on Burgers turbulence and hydrodynamic limit problems. Twenty presentations were given by researchers from US, Germany, Taiwan, Lithuania, Hungary, Japan, China and Russia. Actually, it was a working meeting in the sense that several current research projects in the area were being finished at IMA at the same time.

### *Interaction with leadership and staff of IMA:*

This was an outstanding experience, a model (to be taken back to our home institutions) how well a broad mathematical activity can be organized. Mike Steele did all the background work carefully and selected right people to run individual projects. He was obviously the invaluable idea man for the special year. Avner Friedman has organized a seamless operation at IMA with strong applied mathematics values where scientists from outside mathematics and from industry can collaborate with mathematicians on equal footing. He has to be credited with the outstanding research and pro(intyer)active atmosphere at IMA. The scope of the operation is obviously enlarged compared to what the reporter could observe in the mid-eighties. Willard Miller run the day-to-day operations with complete mastery of and attention to details. Devil in the details was firmly under control.

The staff was friendly, professional and consistently helpful.

### *Research interaction:*

Woyczyński conducted a research program concentrated on the study of Burgers turbulence and on turbulent diffusion, including their applications in oceanography. He pursued a number of collaborative research projects with visitors at IMA and members of the Mathematics Department. Those included joint work

- with Stan Molchanov and Donatas Surgailis on hyperbolic scaling limits in Burgers turbulence,
- with Yiming Hu on shock density analysis for Burgers turbulence,
- with Tadahisa Funaki on nonlocal Burgers equation,
- with Craig Zirbel on rotation number for passive tracer transport for random velocity flows, and,
- with Sasha Saichev on probability distributions of passive tracers in randomly moving media.

The resulting manuscripts have either been submitted for publication or are under preparation. Also an opportunity for perhaps less formal research interaction with several IMA visitors (Mike Steele, Mike Harrison, Frank Kelly, Jim Dai, Ruth Williams, Malyshev, Misha Menshikov, Peter Jagers, Elena Krichagina) and members of the Mathematics and Statistics Departments (Larry Gray, Nick Krylov, Fedorov, Maurey Bramson and old friends like Naresh Jain, John Baxter, Bert Fristedt) and participation in their probability seminar was greatly appreciated.

For a couple of months Woyczyński was also responsible for organization of the twice-weekly IMA Seminar on Applied Probability.

#### *Other workshops at IMA:*

Woyczyński also participated in other workshops at IMA: Mathematical Population Genetics Workshop in January, Stochastic Networks Workshop in March, and Image Models Workshop in May. The first two had exemplary organization and effective leaderships. Their influence extended beyond just the weeks when they were conducted as numerous participants (including the organizers - which was critical) stayed on for several more weeks creating a vibrant research environment. The third workshop felt more like a "one-night stand" variety although there were some good talks given.

#### **Maury Bramson**

Maury Bramson, a visitor of the mathematics department and the IMA for 6 months during the first half of 1994, actively participated in a number of the workshops. In addition to the minisymposium "Phase Transitions in Catalytic Surface Reaction Models" (see below), he was most involved in the workshop "Stochastic Networks". He also participated in "Mathematical Population Genetics", "Stochastic Methods for Nonlinear PDE's" and "Classical and Modern Branching processes." He gave a number of talks in conjunction with the IMA and mathematics department seminars:

"Two Badly Behaved Queueing Networks" - Stochastic Networks Workshop "Nonuniqueness in g-Functions" - Probability Seminar "Ergodicity of Critical Spatial Branching Processes" - Applied Probability Seminar "Survival in One-Dimensional Cellular Automata under Random Perturbations" Pruitt Memorial Symposium

He was quite pleased with the atmosphere at the IMA, and profited from interaction with participants connected with various of the workshops, in particular "Stochastic Networks"

The minisymposium on Phase Transitions in Catalytic Surface Reaction Models took place May 31-June 3, 1994 and was chaired by Maury Bramson and Claudia Neuhauser. This topic has been of considerable interest in the recent physics literature. The basic model consists of two reactants, carbon monoxide and oxygen, which adsorb to a surface, with neighbors of opposite type reacting and desorbing. Depending on the relative concentrations of oxygen and carbon monoxide in the environment, either poisoning of the surface by one of the two reactants occurs, or both reactants continue to coexist. One is interested in understanding the basic mechanisms which are exemplified here. The transition between poisoning and coexistence as the relative concentrations vary also has been studied, and appears to be continuous for the oxygen poisoning-coexistence transition and discontinuous for the carbon monoxide poisoning-coexistence transition. Little has been rigorously established for this latter phenomenon.

The basic model is motivated by the study of the carbon monoxide-oxygen reaction on a single-crystal surface. Although the model leaves out important steps of the actual system, it exhibits much of the steady state behavior occurring in practice. This reaction is among the most extensively studied heterogeneous

catalytic reactions, and is important in automobile-emission control (such as the designing of catalytic converters), among other applications.

Various generalizations and modifications of the basic model are presently being investigated. Many of the active researchers in the mathematical theory of the subject were present at the minisymposium. R. Ziff, one of the originators of the basic model, discussed techniques for testing whether transitions are continuous or discontinuous. D. ben-Avraham, R. Dickman, C. Doering, and J. Evans, members of the mathematical physics community, discussed related mathematical models on phase transitions. C. Neuhauser and G. Swindle, from mathematics, talked on mathematically rigorous results on catalytic surface reactions. M. Baer, Y. Kevrekidis, and G. Yablonskii, from chemical engineering and chemistry, presented experimental results, such as on pattern formation in finite domains.

The members of the minisymposium were pleased with the focus of the conference. There were numerous impromptu discussions among participants. Various comments were volunteered at the end of the minisymposium on the friendly and constructive atmosphere at the IMA.

### Claudia Neuhauser

Claudia Neuhauser visited the IMA from January 1, 1994 to June 31, 1994. She participated in a variety of workshops and seminars, in particular, she, together with Maury Bramson, organized a minisymposium on catalytic surface reactions where she presented a talk on "Coexistence for Catalytic Surface Reactions." Furthermore, a paper she co-authored, was presented by Stephen Krone at the Mathematical Population Genetics workshop. She attended the workshops Stochastic Networks, Image Models and their Speech Model Cousins and Classical and Modern Branching Processes. She gave a talk in the Applied Probability Seminar on "Distributional Results for Sequence Comparisons." In addition to these activities, she continued to work on various projects.

**Krishna B. Athreya** co-organized the June 1994 workshop on *Classical and Modern Branching Processes*, and was at the IMA for one month. Here is his report on the workshop:

As the final part of its year on Emerging Applications of Probability, the IMA arranged a workshop on Classical and Modern Branching Processes during June 13-17, 1994 in Minneapolis. The organizers were Krishna B. Athreya of Iowa State University and Peter Jagers of the Chalmers University of Technology (Sweden). The workshop was indeed a fitting finale to a full year of probability theory and its applications. It brought together some forty-seven researchers from sixteen countries. Some of the exciting new developments included applications of branching processes to biology, geophysics and computer science, new approaches via measures on random trees, large-deviation aspects of explosive branching, and general branching with dependencies. The excellent research atmosphere at the IMA and the very friendly and helpful staff contributed greatly to the success of not only the workshop but also the two months preceding it, during which a number of visitors came for varying periods of time, leading to very fruitful interaction among researchers. The workshop proceedings will be published by Springer-Verlag in their IMA series, with Athreya and Jagers as editors.

The topics covered included a broad spectrum of branching processes. A general classification goes as follows:

- a) New approaches to the limit theory of branching processes via measures on trees (Lyons, Pemantle, Peres);
- b) Branching random walks, diffusions, Markov processes (Biggins, Wakolbinger, Dawson, Dekking, Gorostiza, Sagitov, Chauvin);
- c) Branching processes and random trees (Vatutin, Kersting, Joffe, Roualt);
- d) Applications to biology, geophysics, and computer science (Kimmel, Heyde, Klebaner, Waymire, Zhang);
- e) Large deviation problems (Athreya, Vidyashankar, Ney);

- f) General branching processes with dependence (Jagers, Olofsson);
- g) Inference and distributional results (Cohn, Pakes, Yanev, Dion, Rahimov).

## POSTDOCTORAL MEMBERS

### Martin Hildebrand

While at the I.M.A., I have been involved with a number of activities.

I have continued to get some results in my research. Some preprints are a result of this research; in particular, the I.M.A. preprints "Enumeration and Random Random Walks on Finite Groups" (joint with C. Dou) and "Random Walks on Random Regular Simple Graphs" contain results which I hope to have published. The former preprint contains the answer to a question I posed in the open problems session of the October workshop. The question concerned random walks on finite groups. If a random walk is based on a random choice of  $k := \lfloor (\log n)^a \rfloor$  elements where  $n = |G|$  and  $a > 1$ , how long does it usually take the walk to get "close to uniform" on the group  $G$ ? The result in the paper shows that

$$\frac{a}{a-1} \frac{\log n}{\log k} (1 + \epsilon)$$

steps are enough with  $\epsilon > 0$ . For certain groups (abelian and non-abelian), this result is the best possible; the factor  $a/(a-1)$  is essential there. The work on random walks on random regular simple graphs examines the case where the degree  $d := \lfloor (\log n)^a \rfloor$  where  $n$  is the number of vertices and  $a \geq 2$ . For most graphs,  $(\log n / \log d)(1 + \epsilon)$  steps are enough to make the random walk close to uniformly distributed on the vertices for most such graphs. Although there are results for eigenvalues of random regular graphs, these results use a model of random graphs where the graph is very likely not simple. While the eigenvalues tell some things about random walks on that model of random graphs, the results are not obviously transferable to the model using simple graphs.

One smaller result, a simple counterexample to a conjecture of C. Dou, has lead to a question which is intriguing but remains open. This question is as follows: Let  $S_1$  and  $S_2$  be disjoint subsets of a finite group  $G$ . Let  $T = S_1 \cup S_2$ . Let  $P$  be uniform on  $S_1$ , let  $Q$  be uniform  $S_2$ , and let  $R$  be uniform on  $T$ . (In all cases, these probabilities will be 0 outside the specified subsets of  $G$ .) Is  $\|R^{*m} - U\| \leq \max(\|P^{*m} - U\|, \|Q^{*m} - U\|)$  for all  $m$ ? ( $P^{*m}$  is the probability distribution of the sum of  $m$  i.i.d. random variables each with probability distribution  $P$ .  $\|P - U\|$  is the variation distance of a probability distribution  $P$  from the uniform distribution  $U$  and is as described in the book of P. Diaconis.) The counterexample to the conjecture shows that in some cases,  $\|R^{*m} - U\| > \min(\|P^{*m} - U\|, \|Q^{*m} - U\|)$ .

In another result, I have developed proofs of special cases of a combinatorial conjecture of R. Simion; these proofs have been alluded to in a preprint of B. Sagan. This conjecture concerns the number of lattice paths in rectangles where the Ferrers diagram of a given partition has been removed from the rectangle.

This year I have had several papers accepted for publication. "Random Walks Supported on Random Points of  $\mathbf{Z}/n\mathbf{Z}$ " has been accepted by *Probability Theory and Related Fields*. "Random Processes of the Form  $X_{n+1} = a_n X_n + b_n \pmod{p}$  Where  $b_n$  Takes on a Single Value" is to appear in the Proceedings of the I.M.A. Workshop Random Discrete Structures. "Log concavity of the Number of Paths from the Origin to Points Along the Line  $(a, b, c, d) + t(1, -1, 1, -1)$ ," a joint paper with J. Starkweather, has been accepted by *Ars Combinatoria*.

I have given a number of talks this year. I gave 3 talks in the I.M.A. post-doc seminar, 3 talks in the Combinatorics Seminar, and 1 talk in the Applied Probability Theory seminar. I also gave talks at the American Mathematical Society meetings in Cincinnati in January and in Minneapolis in August. After my arrival at the I.M.A. in the summer of '93, I gave a talk at Drexel University, and during the winter, I gave a talk at the University of Wisconsin - Madison. I also gave talks at several job interviews.

I have interacted with a number of I.M.A. visitors. Of note, interactions with Persi Diaconis led to some early results which led to some results in the preprint with C. Dou. I talked with Robin Pemantle about some problems, and I talked with Joel Spencer and David Aldous about some of my results. I talked with Mike Steele about various things. I have also corresponded via e-mail with some of the I.M.A.

visitors. I corresponded with Svante Janson about some results to see if he knew about them previously. I have corresponded some with Daming Xu about various results in random walks on finite groups. I have corresponded with Anant Godbole on a heuristic which rewords a question in Ramsey theory; the rewording has interested both of us. I also had both some conversations and e-mail correspondence with David Wilson; this interaction has been beneficial for both of us.

I have also done some service activities for the I.M.A. and other parts of the mathematical community. I organized the I.M.A. post-doc seminar and encouraged various people to speak in it; with a relatively small number of post-docs and with ample opportunity for post-docs to speak in other seminars, this organization can be a challenge. I chaired one session at the November workshop, and I chaired part of a session of contributed papers at the Minneapolis A.M.S. meetings. I have also refereed a number of papers and grant applications.

**Timo Seppäläinen** was a Postdoctoral Member of the IMA for the 1993-94 Probability year. In the following report, he only describes work that has led or will lead to publications. In addition, he had constant interaction with visitors of the IMA and members of the School of Mathematics faculty in seminars, workshops, and various graduate courses:

During Fall, 1993 I worked on 2 projects:

*The statistical mechanics of disordered systems*

This is an ongoing project whose origins are in my 1991 thesis on the large-deviation theory of non-stationary stochastic processes. During the fall term I completed a phase of it that involves developing large deviation techniques for calculating critical exponents for a mean field model with disorder. This work appeared as an IMA Preprint and will appear in *Prob. Theory Rel. Fields*.

*Large deviations for Brownian motion.*

This is a joint project with Peter March (Ohio State Univ.) About the large-deviation theory of Brownian motion under a natural multiplicative group action that leaves Wiener measure invariant. As an application we can study large deviations from the so-called almost-everywhere central limit theorem. This project still needs a few technical details before it is ready for publication.

Among the Fall term visitors I had particularly frequent interaction with W. Bryc from the University of Cincinnati, who also, works on large deviations. We discussed his joint project with A. Dembo on the large deviations of Gaussian processes, which subsequently appeared as an IMA Preprint.

The Winter and Spring of 1994 were mostly spent on one project, developing and studying a stochastic dynamical system that is macroscopically governed by the porous medium equation. This means that the stochastic process is interpreted as a microscopic description of the system, and its macroscopic description is revealed by a scaling limit under which the stochastic evolution converges to a deterministic function that obeys the porous medium equation. The work was joint with Michael Ekhaus, at the time a graduate student in the School of Mathematics, Instrumental for its successful completion were discussions with D.G. Aronson from the School of Mathematics and with S.R.S. Varadhan of Courant Institute who visited the School both in the winter and spring. I am presently writing a preprint on this work.

In June 1 I attended the AMS Summer Seminar on Applied Mathematics at MSRI, on partial differential equations and probability theory. The remainder of the Summer of 1994 was spent in extending some earlier work on a relative entropy function appropriate for disordered models, and in revising and improving work that had been refereed for journals.

### J. Biggins

I spent the greater part of my time at the IMA working on two projects. Both concerned my main area of interest, which is the branching random walk. This is a model in which a parent reproduces to give daughters which are displaced from their parent's position. Each daughter then reproduces in a similar way, and so on. Thus, this can be thought of as a model for a population that reproduces asexually with children moving away from their parent's position.

It is natural to wonder how fast this growing population spreads out. This issue was addressed in a series of papers in the 1970's; under suitable conditions on the reproduction and displacement mechanism, the process spreads out linearly with time, and the rate of spread can be calculated. Complete (first-order)

results on the speed are known for the discrete-time model (where time is the same as generation) but the theory for continuous-time models was less fully-developed. More recently, these results have found several applications. Interestingly, the nicest applications are not in demography (as might be expected from the description given) but in the theory of algorithms. Stimulated by the application the known results had found, I spent some time producing a satisfactory continuous-time theory and looking at some applications of it. Two new features are possible, and were included, in the continuous-time model: people are allowed to move around; and a child's initial displacement from her mother and her mother's age at the child's birth may be correlated. This work was both undertaken and written up (as two papers) whilst at the IMA.

*How fast does a general branching random walk spread?* should appear in an IMA proceedings volume, edited by K.B. Athreya and P. Jagers.

*The growth and spread of the general branching random walk* has been submitted to *Annals of Applied Probability*.

This work has reached a natural end-point. However there are several other cases where fuller results are available for the discrete-time model than for the continuous-time one that it would be interesting to try to cast into the framework developed. Also, there is a significant literature on deterministic (as opposed to stochastic) models for growth and spread, usually motivated by biology. Whilst at the IMA I made a start on examining the relationship between this literature and the results I obtained, and mentioned this in the first of the two reports above, but there is undoubtedly more to do.

The second main project I worked on concerned a question left open by recent work on what are called Lindley-type equations in the branching random walk. These arise when the all-time maximum displacement in branching random walk on the real line is considered. The distribution of this satisfies a certain recursion which is known to have more than one solution. I have been working on the problem of identifying the solution set, and interpreting it. The project is well advanced (with 20-odd pages already written) and I hope that an essentially complete solution will result. At the IMA I had the opportunity to discuss this project (and the manuscript) with one of the main authors of the previous papers on the matter (Yurii Suhov), who was excited by the developments. There are not-fully-understood connections between this work and the existence of travelling-wave solutions to certain deterministic equations.

A third project, with Krishna Athreya, also received some attention. This was stimulated by a paper in theoretical computer science considering how a set of substitution rules cause the number of available sentences to grow when the rule is applied repeatedly. We have obtained more precise results than those presently available for this enumeration problem, but our study does not (yet?) cover all cases. Since returning to Sheffield I have discovered that the same kind of enumeration problem has arisen in the study of the Gibbs sampler on structured genetic pedigrees.

I made a number of useful new contacts and renewed some old ones at the IMA. Probably the most significant new one for me was meeting Ed Waymire, and learning more of the work on Random Cascades (which are Branching Random Walks by another name). The meeting on Stochastic Modelling of Geophysical Systems (or some similar name!) was also important here. I was both interested and excited to see the model I had thought about a lot (albeit under another name) being used by several of the speakers in practical investigations of geophysical phenomena. Discovering this work has introduced new questions into my research agenda. Where this will lead remains to be seen, but I am optimistic that something will come of it.

**Nicholas Krylov** is on the Mathematics faculty at the University of Minnesota; he participated in the 1993-94 activities at the IMA, and reports as follows:

I participated in the IMA activities for 1993-94, when the topic was, "Emerging Applications of Probability" and found that all the time I spent on IMA activities was worthwhile, and that some of the research contacts I made will turn out to be fruitful.

The most interesting for me were workshops organized by Ruth Williams, Stanislaw Molchanov and

Wojbor Woyczynksi.

I delivered a talk at the period of concentration organized by Wojbor Woyczynksi: "Stochastic Problems for nonlinear partial differential equations."

Also I wrote an article with the same title and the article is accepted for publication in SIAM J. Math. Anal.

## 5. Research Reports by Long Term Visitors: 1994-1995 Waves and Scattering

### William Symes

I was very pleased to learn that ONR is funding the March workshop, and of course would like to help that connection stay healthy. I am on the road at the moment and only just yesterday got my email straightened out. So I hope this is not too late to be useful.

Overall, I think that the subject is due for a large infusion of insight from science and engineering applications. There are certainly interesting mathematical developments underway, for example the large current interest in so called downward continuation methods for elliptic problems (Cheney and Sylvester spoke about this), exploration of bounded variation constraints (Acar and Vogel, Santosa and Dobson), better understanding of nonlinear regularization (Engl, Neubauer, and the Austrian school, unfortunately not represented at the workshop), and vastly improved theory of direct and inverse problems for 1D waves in very heterogeneous media (Papanicolaou and collaborators). However, in my opinion the most interesting work for the next while will be very close to applications.

I will mention a couple of areas that seem to me ripe for progress in the near future. Obviously, my own interests will color my choices, but, I think not too much.

Theoretical reflection seismology has benefitted greatly from the "scale - separated" viewpoint, which partly linearized the dependence of the wavefield on the coefficients of the wave equation. Short scale coefficient components are treated as first order perturbations about smooth background fields. The latter (notably wave velocities) still have nonlinear influence. One could say that this approximation is entirely responsible for the existence of reflection seismology as a discipline, and it's not yet tapped out. Over the next couple of years however, I expect the theory of this approximation will finally approach completion. Beylkin, Burridge, and Rakesh supplied the proper theoretical venue (microlocal analysis) in the 1980's, and treated the local version of this description of scattering. Recently, Smit and his coworkers, Uhlmann, and Nolan and myself, have come close to completing the global picture, with a complete account of scattering in the vicinity of caustics. Chavent, Gerard Herman, and my group have made considerable progress in devising algorithms to estimate the smooth background parameters as well. Most of this work so far has concerned the simplest models, but in the next few years all of this will be extended to quite realistic models of seismic wave propagation in the upper crust and radar wave propagation in the near surface, with attenuation and anisotropy properly accounted for (deHoop, Smit, and Burridge have laid the groundwork for proper treatment of anisotropy). Chavent has made some progress in developing a sound theoretical basis for scale separation, but more work is needed.

I wish to emphasize the nature of this work. It is quite rigorous, relying extensively on modern analysis of PDEs. On the other hand its intellectual context is very much that of the application, in this case exploration geophysics. It is oriented toward computation, and will be applied to field situations with little delay.

Two very different talks at the workshop represented another trend which I believe will be elaborated extensively in the next few years: analysis of information content of data. Stark's work on reliability of whole earth tomography, and Foster and key's presentation of the "mudrock line" interpretation of amplitude-versus-offset analysis of reflection seismograms, both show how careful analysis can distinguish information from artifact in linear inverse problems. I expect this type of analysis to become more useable, with better displays of its conclusions to enhance interpretation, and more refined, accounting for nonlinearity. Every applied inverse problem will benefit from these developments, with ocean acoustics, medical imaging, and nondestructive evaluation particularly ripe.

Papanicolaou and his associates have developed a very extensive theory of bulk behaviour of plane waves in finely laminated materials, with considerable predictive capacity. Most Earth materials, for example,

are highly disordered but not laminated, or only approximately laminated. The bulk behaviour of waves in nonlaminated materials, for instance apparent attenuation and dispersion of pulses due to scattering, is qualitatively different from that in laminated materials, due to absence of resonances. Progress in understanding these phenomena is very important, for example in permitting the theory of reflected wave inversion to move beyond linearization based on scale separation. The development of magnetic resonance imaging of rocks and thin section digitization, and of microscopic porous fluid dynamics techniques, should permit both computational and laboratory tests of bulk wave theory in the next few years.

So these are my "picks." You would be entirely within your rights to wave them in my face in five years.

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### George Papanicolaou

First, I would like to point out that the March 6-17, 1995 IMA Workshops on 'Inverse Problems in Wave Propagation' was enormously successful and, in my view, the best part of our program on 'Waves and Scattering' for the 1994-95 year at IMA. I feel free to say this as Chairman of the organizing committee for the program because the winter period on 'Inverse Problems', and the March workshop in particular, were conceived and organized by Bill Symes almost entirely. The rest of the committee was happy to go along with his ideas and proposals. The program was truly interdisciplinary, original and exciting both for the mathematics and its applications.

The most pressing issue in Inverse Problems in Wave Propagation is to bridge the gap between theoretical studies of inverse problems and the mostly empirical methods that are used in imaging. To paraphrase Jon Claerbout, it was an eye opening experience for him back in the late sixties and early seventies to move from the study of inverse problems in seismology as they came up in the nuclear treaty verification context to the world of seismic imaging as was (and still is) practiced by the hydrocarbon exploration industry. Seismic imaging deals with the underlying inverse problem in a very naive and oversimplified way but is very sophisticated in the way it deals with the available data (time traces of amplitude at different offsets, different source locations, etc.). Relatively simple amplitude versus offset analysis and transformation of the data gives a crude but frequently quite adequate picture of the structure of the velocity as a function of position down to depths of a five kilometers or so. The best mathematical studies to date (Bleistein's group at the Colorado School of Mines and Symes' group at Rice) do not really explain why this simple minded analysis works as well as it does and, more important, how the theory can point the direction for improvements. But their efforts and their work are taken very seriously in the seismic exploration community and this is a major achievement for theoreticians. Other theoreticians working successfully in this area are Burridge, deHoop, Beylkin and Cheney.

To finish Claerbout's remarks I should also point out that in nonproliferation seismology the theoreticians did succeed in getting accurate yields by the mid-eighties, but they did it in an entirely empirical way. Contrary to what is happening in exploration seismology, there are almost no theoreticians now working on getting a good basic understanding of why the empirical theory works (and how to improve it!). There are lots of really interesting theoretical problems there that have to do with mode coupling of elastic waves by inhomogeneities over long distances near the surface. The role of radiative transport theory in seismology is not well understood today, despite the fact that it has been used for more than ten years now (by Wu, Sato, Aki and others). But as in exploration seismology most researchers are, at present, heavily involved with the management (data compression, archiving protocols) of very large data sets.

I should mention that velocity estimation in seismic exploration, a basic statistical problem, has received very little attention from first principles except in my own work with Kohler, White, Burridge and others. Despite the fact that we have solved and implemented almost completely several idealized but quite relevant direct and inverse seismic problems that are dominated by noise effects (as is the case in the real world), we have not succeeded in turning our ideas and results into useful tools for the exploration seismologists. There is gap both in applicability and in complexity between what we do and what the exploration people are prepared to do (with their much larger data sets, etc). A lot more must be done in understanding the role of noise in seismic data. It does increase the complexity of the mathematical problems but it also provides a natural mechanism for stabilizing some inherently unstable inversion processes.

Seismic applications are by far the biggest clients of research in 'Inverse Problems in Wave Propagation'. However, in the last ten to fifteen years, ocean acoustic tomography has developed rapidly and has posed

several versions of inverse problems that are interesting and subtle. Much of the field work is based on travel-time tomography with a least-squares inversion. In my view, the most interesting theoretical ideas here are in the work of Berryman and his collaborators who have used the variational structure of travel times (Fermat's principle) to get a-priori feasibility bounds for the inversion results. This has the very desirable effect of making least-squares inversion much more stable and more rapidly converging. Berryman's work is not as well known as it should be, I believe, both by theoreticians and practitioners. When noise effects are added from first principles we have some very exciting mathematical problems coming up that need detailed study.

### Margaret Cheney

Spending my sabbatical at the IMA, for its special year on waves and scattering, was very constructive for me. I learned a great deal by attending most of the talks at most of the tutorials and workshops. I had many useful conversations with visitors to the IMA. I worked on the following problems:

#### Finishing the Layer-Stripping Paper.

I finished up my paper with David Isaacson on a layer-stripping approach to reconstructing a perturbed dissipative half-space. This paper appeared in the IMA preprint series, and was published in *Inverse Problems*. While at the IMA, I gave a number of talks about this work.

#### Consulting for Endocardial Solutions

The presence of both myself and Eric Voth at the IMA led me to a formal consulting arrangement with Endocardial Solutions, a small start-up company. This company is attempting to develop an electrical system for treating cardiac arrhythmias without open-heart surgery. They do this by passing into the heart a probe covered with electrodes. The success of the system depends on using the probe to sense the location of conduction abnormalities. This involves the following three main mathematical problems 1) Given voltage measurements on the probe due to current sources on the probe, determine the location of the endocardium; 2) Given the location of the endocardium, map the naturally occurring electrical potentials on the endocardium; 3) Use the probe to determine the location of an additional ablation catheter.

These problems are all ill-posed inverse problems reminiscent of the electrical impedance imaging problem I work on at Rensselaer.

#### Modelling an Anesthesia Breathing System.

I worked with Paul Bigeleisen, a staff anesthesiologist at St. Paul-Ramsey Medical Center, in developing a mathematical model of the breathing system used to administer anesthesia. This problem is very much in the spirit of the IMA Industrial Mathematics program, in that it involved: 1) a practical, real-world problem, 2) close collaboration between physician and mathematician, 3) modelling work, and 4) experiments.

The problem is described in the following excerpt from the introduction to our paper.

A critical time in the administration of anesthesia is the period called induction when the patient has received a drug dose sufficient to paralyze the muscles of respiration, but before tracheal intubation and mechanical ventilation has commenced. So that the anesthetist will have the most possible time to accomplish intubation before the patient become hypoxic, the patient should start with the greatest possible oxygen reservoir. In particular, the nitrogen normally in the lungs should be replaced with oxygen before induction. This process, called denitrogenation, is accomplished a few minutes before the anesthetic drugs are given by having the patient breathe pure oxygen through a mask connected to the breathing circuit. This circuit is also connected to the anesthesia vapor machine and is used to administer anesthetic gases after induction and intubation. There has been considerable debate within the anesthesia community as to the most efficient method to accomplish denitrogenation. In 1981, Gold, using blood gas samples, showed that four deep breaths from a standard circle system produced an oxygen concentration in the blood equivalent to that accomplished by breathing normally for five minutes from the same circle system. Thereafter, it became standard practice to have the

patient take four deep breaths of pure oxygen prior to the induction of anesthesia. More recent studies, using pulse oxymetry and in line mass spectrometry, have suggested that four deep breathes are not as efficient at denitrogenation as normal tidal breathing for three minutes. The latter authors suggest, without evidence, that greater rebreathing of nitrogen from the circle system when large breaths are taken, is the cause for inefficient denitrogenation. With rebreathing, the patient is actually recycling part of his/her own breath, including nitrogen, through the circle system. This effect slows down the elimination of nitrogen from the lungs.

In order to determine the most efficient method of denitrogenation, we developed a model of the circle system and compared it to experimental data using the authors as subjects.

Our mathematical model consisted of a system of 8 coupled differential-delay equations. We implemented this model in Matlab. We have nearly completed a paper based on this work.

#### Processing of Radar Data

I discussed with a number of visitors my ideas for using a geometrical optics expansion to recover the surface electrical properties of a material from the first reflection of a radar pulse. Greg Beylkin suggested that a certain matrix in my scheme might be ill-conditioned, which led me to a method for overcoming this difficulty. Ingrid Daubechies gave me some advice on windowing techniques for signal processing.

#### Wave Propagation in Random Media

While listening to talks on wave propagation in random media, I started wondering if the method of multiple scales could be used to study the propagation of electromagnetic waves in sea ice. This is a difficult wave propagation problem, because sea ice contains a multitude of pockets of brine and air, which are strong scatterers of electromagnetic waves. I discussed this problem with a variety of visitors, who uniformly found the problem very interesting but did not know how to solve it. I am continuing to think about this problem back here at RPI, and am discussing it with Julian Cole.

#### Other Connections with Industry

**Doug Huntley**, one of the industrial postdocs, arranged for me to give a talk at 3M. This eventually led to an arrangement between 3M and my impedance imaging research group at Rensselaer, in which 3M supplies us with electrodes. I discussed with Keith Kastella, a staff scientist at Unisys, the possibilities for using electrical impedance imaging to determine the extent of ground contamination at waste disposal sites. Our discussions led to an improvement in the algorithm he was developing. I met with representatives of Renaissance Technologies to discuss similarities between the Rensselaer impedance imaging device and their IQ system for assessing cardiac function. The IQ system uses the bulk electrical resistance of the torso, together with sophisticated signal processing techniques, to recover information about cardiac function.

**Tuncay Aktosun** of the Mathematics Department of North Dakota State University visited the IMA during January 1-March 31, 1995. He interacted with many scientists visiting the IMA and benefited very much from his visit.

Dr. Aktosun's recent work involves the scattering and inverse scattering problems associated with quantum mechanics and wave propagation. In particular, he works on the Schrödinger equation and the wave equation with variable speed. The inverse scattering problem consists of obtaining a coefficient in these equations in terms of an appropriate set of scattering data. It is possible to combine these two equations to get a generalized Schrödinger equation: for the special choice of the coefficients this generalized equation reduces to the Schrödinger equation and to the frequency-domain version of the wave equation with variable speed, respectively. The generalized equation itself governs the wave propagation in a nonhomogeneous medium when there is a restoring force present in the medium.

While at the IMA Dr. Aktosun worked on both one dimensional and three dimensional versions of these equations. The wavefield of the frequency-domain wave equation can be obtained by solving a Riemann-Hilbert problem; however, the solution of the latter problem requires that one knows the large (complex) frequency asymptotics of the wavefield. In three dimensions these asymptotics are not yet known and the knowledge of these asymptotics will be a giant step in the recovery of the wavespeed in a nonhomogeneous, nonspherically symmetric medium. Dr. Aktosun learned from Michael E. Taylor at the IMA that, under

certain restrictions, the large (real) frequency asymptotics of the wavefield grow no faster than the  $3/2$  power of the frequency. Currently, he is working on the generalization of this result to complex frequencies.

In a one-dimensional medium where the properties change abruptly at a finite number of interfaces, while at the IMA, Dr. Aktosun developed an exact procedure to obtain the discontinuities from the large frequency asymptotics of a reflection coefficient. The results obtained are used in a method currently being developed to obtain the full wavespeed from the reflection coefficient.

Dr. Aktosun interacted with Paul Sacks regarding the recovery of the potential of the Schrödinger equation when the phase of the reflection coefficient is unknown. He communicates with Dr. Sacks on this and various other problems of mutual interest; Dr. Sacks was also visiting the IMA during January-March, 1995.

*The Preprints:*

- [1] T. Aktosun, M. Klaus, and C. van der Mee, Recovery of discontinuities in a nonhomogeneous medium, IMA May, 1995, preprint #1310.
- [2] T. Aktosun, Inverse Schrödinger scattering on the line with partial knowledge of the potential, IMA August, 1995 preprint.
- [3] T. Aktosun and C. van der Mee, partition of the potential of the one-dimensional Schrödinger equation, IMA August, 1995 preprint.

**Paul Sacks** While visiting the IMA, January-March, 1995

1) In joint work with Rakesh (University of Delaware) we have proved a global uniqueness theorem for a one dimensional inverse problem for the variable impedance wave equation, in which finite time transmission data is the given information. This work also presents a numerical solution method.

2) Joint work with J. McLaughlin (R.P.I.) was continued, on the subject of inverse problems for acoustic media involving interior transmission eigenvalues. This kind of problem arises as a special case of the more general and well-known problem of determining the sound speed distribution in  $\mathbb{R}^3$  from knowledge of far-field patterns. Our work is mainly concerned with uniqueness and non-uniqueness theorems, and also with the development of numerical methods.

3) The opportunity to meet with V. Yakhno (Siberian Academy of Sciences, Novosibirsk, Russia) has led to the formulation of plans for more long term collaboration on certain inverse problems in linear elasticity.

General Remarks about Contributions of the IMA Visitors:

1) The tutorial lecturers gave a great many interesting and enlightening talks. They were very well appreciated by the IMA audience in residence at the time.

2) The main workshop in March brought together researchers from many different scientific and engineering disciplines who ordinarily would have little contact with each other. For the mathematicians attending the conference it was a fairly unique opportunity to learn about and discuss some very practical and important inverse problems arising in a number of scientific areas.

**Michael Taylor**

Here's a report on some IMA activities I'm aware of. I hope it will be useful.

In the workshop on inverse problems, the connection between stabilization of various ill-posed problems and the Carleman estimates arising in unique continuation theorems was brought out clearly, particularly by V. Isakov. A month later, C. Bardos brought out similar connections between unique continuation and control theory. Both made use of recent results of D. Tataru.

A different sort of microlocal attack on stabilization of a class of inverse boundary problems was made by M. Taylor, in work completed at the IMA. The work chiefly dealt with linearized inverse problems. Conversations between Taylor and W. Symes have suggested directions for further progress on the nonlinear problem. Work here is still in progress.

C. Liu reported on progress on inverse problems for scattering of waves by Lipschitz obstacles, joint work with A. Nachman. One consequence of this work bore on the famous problem of uniqueness of a obstacle given scattering data at one frequency, and one incident direction. They showed that, at most, finitely many Lipschitz obstacles can have such common data. One offshoot arose in conversations between

Liu and M. Taylor: one can identify a sphere in  $IR^3$ , given scattering data at one frequency, from any two linearly independent incident directions.

Work on Lipschitz boundaries connected with other work, both at the IMA and in the mathematics department at Minnesota, particularly works of E. Fabes and of M. Mitrea. The latter, in joint work with J. Pipher, examined direct scattering problems of electromagnetic waves by Lipschitz boundaries. This work had further contact with work of M. Taylor, on the Euler and Navier-Stokes equations for incompressible fluids. Work on this is still in progress; one direction involves a possible simplification and further extension of recent work of P. Deuring and W. von Wahl, on the Navier-Stokes equations in a Lipschitz domain. Taylor reported on progress on the Euler equations in the department's PDE seminar.

M. Cheney presented work on layer stripping as a technique in inverse problems. This involves examination of the Dirichlet-to-Neumann map. In conversations with M. Taylor it was seen that this technique has interesting similarities with analysis brought to bear on the  $\bar{\partial}$ -Neumann problem.

In the workshop on singularities and oscillations, there was particular interest in the behavior of solutions to semilinear wave equations. J. Rauch described some models for the propagation of electromagnetic waves, interacting with a semiconducting medium, which might better accommodate observed phenomena of dispersion than standard models. Several participants, including H. Lindblad and Tahyildar-Zadeh, have been studying semilinear wave equations whose nonlinear terms satisfy a 'null condition.' One significant example is the wave map equation, a Lorentz analogue of the harmonic map equation, of interest to physicists because of its role in 'nonlinear  $\sigma$ -models.' Wave map equations belong to a class for which classical solutions can blow up in finite time. S. Alinhac spoke on a general study of mechanisms for blow-up of solutions, a subject on which S. Kichenassamy has also been active.

An interesting study of some quasilinear systems was made by R. Rosales. He gave a talk on the difference between long-time behavior of solutions to the  $2 \times 2$  and the  $3 \times 3$  systems of compressible fluid dynamics, in one space variable, including newly discovered families of persisting waves for the  $3 \times 3$  system.

In the workshop on quasiclassical methods, connections with quantum statistical mechanics were noteworthy, and played an important role in the tutorial of B. Helffer. The message is that 'classical' manifestations of physical phenomena, obeying underlying laws of quantum mechanics, arise not only because one is looking on a dimensional scale in which Planck's constant is small, but also because one is looking at the interaction of a large number of particles. An IMA visitor, V. Jaksic, was working on the interaction of particles with a 'heat bath,' both classical and quantum mechanical, and was pursuing a dynamical theory of Brownian motion using such a model.

### David Finch

During the winter of 1995, while a visitor at the IMA, I began again to study a problem which had intrigued me some years ago. In a mathematical model of single photon emission tomography, measurements exterior to a body  $\Omega \subset R^2$  with (variable) attenuation coefficient  $a$  and distribution of radioactive emitters  $f$  can be represented by an integral transform along directed lines  $L$  given by

$$R_a f(L) = \int_L f(x) \mu(x, \theta) dx$$

where the integral is taken with respect to arc length,  $\theta$  is the direction of  $L$  as a point in the unit sphere, and  $\mu(x, \theta)$  is the exponential of the negative of the integral of  $a$  over the ray starting at  $x$  with direction  $\theta$ . Many questions about this generalized Radon transform remain open: two which I have been interested in are uniqueness when  $a$  is known and the bolder question, much studied over a period of 10 years by Natterer, of uniqueness of both  $a$  and  $f$  from full measurement over all lines passing through the support of the unknown functions. For the second question, there is a known class of counterexamples. Namely, if  $\Omega$  is the unit disk and both  $a$  and  $f$  are radial functions the  $R_a f$  is radial, but it is known that such a function is in the range of the operator  $R_a$  for  $a = 0$ . The project I began at the IMA is to look at the linearization of this question: that is to investigate uniqueness questions about the differential of the map  $(a, f) \rightarrow R_a f$ . Even for the simplest case, when  $a = 0$  so that one has the classical x-ray transform, the

question of injectivity of the differential leads to non-standard problems in integral geometry, which as yet are unsolved.

Two germs of projects, begun in conversations with other visitors at the IMA, but not much advanced, are the analysis of the classical Radon transform in the space  $BV$ , which would seem to better model the kinds of density functions one might see in applications, and the application of some uniqueness theorems of integral geometry to some special inverse problems for the wave equation. There has been much work done on the latter by the Novosibirsk school: the question is to see whether integral geometry can substitute for results in partial differential equations.

### Rakesh

During my stay at the IMA from January 7, 1995–April 15, 1995, I worked on two problems

- Inversion from transmission data for the one dimensional wave equation
- Inversion from point source data for a multidimensional inverse problem for the wave equation

The first problem was solved jointly with Paul Sacks and the article based on this has been submitted to *Wave Motion*. Conversations with David Finch led to results on subproblems of the second problem and discussions with Walter Strauss and Daniel Tataru have resulted in ideas on how to tackle the full problem. In addition, I attended the tutorial and the workshop on Nonlinear Waves in April and I think the tutorials of Beals and Taylor will pay dividends in the future. Below, I give a more detailed description of the two problems I studied.

#### *Inversion from transmission data for the one dimensional wave equation*

Suppose  $\eta(x)$  is a positive function in  $W^1\text{-}infty(0, \infty)$  (i.e. one bounded derivative), is constant for  $x > X$  for some known positive number  $X$ . Consider the initial boundary value problem

$$\begin{aligned}\eta(x)u_t(x, t) - (\eta(x)u_x(x, t))x &= 0, & 0 \leq x, t \in R \\ (u(x, t) &= 0, & 0 \leq x, t < 0 \\ u_x(0, t) &= \delta(t), & t \in R\end{aligned}$$

where  $\delta$  is the usual Dirac delta function. From causality,  $u$  is zero for  $t < x$ . Further, using energy estimates, one may show that the above initial boundary value problem has a unique solution which is locally  $H^1$  in the region  $t \geq 0$ . We were interested in the inverse problem of recovering  $\eta$  from transmission data  $u(X, \cdot) - X$  fixed. We proved

Assume  $\log \eta \in W^1\infty(0, 2X)$ , and  $\eta$  is constant for  $X \leq x \leq 2X$ . Then  $\frac{\eta(x)}{\eta(0)}$  is uniquely determined by  $u(X, t)$ ,  $X \leq t \leq 3X$ .

Further, our proof was constructive, and we tested a numerical implementation of our algorithm.

#### *Inversion from Point Source Data for a multidimensional inverse problem for the wave equation*

Suppose  $q(x)$  is a smooth function on  $R^3$  with  $q$  zero outside the  $a$  ball of radius  $\rho$ . Consider the initial value problem

$$\begin{aligned}u_{tt} - \Delta_x U + q(x)u &= \delta(x, t) & x \in R^3, & t \in R \\ u &= 0 & \text{for} & t < 0\end{aligned}$$

Using standard techniques ones can show that this initial value problem has a unique smooth solution over the region  $t > |x|$  - the solution is zero on  $t < |x|$ . Our goal was to study the inverse problem - given  $u(x, t)$  on  $\{x : |x| = \rho\} \times [\rho, 3\rho]$ , determine  $q$ .

I can now show that there is uniqueness for the inverse problem provided we restrict our attention to the class of functions  $q$  which have small  $L^\infty$  norms. However, I wish to prove a similar result without any restrictions on the size of  $q$ . I have some ideas on how to tackle the general problem based on the work of Tataru but this is a long term project.

### Valery E. Grikurov

#### *Some Aspects of Soliton Behaviour in a One-Dimensional Nonintegrable Schrödinger Model*

The propagation of lasers through nonlinear optic media is described by the nonlinear Schrödinger (NLS) equation

$$i\psi_t = \Delta\psi + F(|\psi|^2)\psi, \quad (1)$$

where “time”  $t$  is a coordinate along the direction of propagation and  $\Delta$  is the Laplacian with respect to transverse variables. The most popular version of the equation (1) is the one-dimensional (1D) “cubic” NLS:  $F(\xi) = \pm\xi$ . The case with the minus sign is called “focusing”: the nonlinearity tends to compress a wavetrain. Due to the balance of dispersion and the compression, the soliton, that is, the localized finite energy solution, exists for both the 1D and the multidimensional focusing NLS equation. In the 1D case, the “cubic” NLS equation can be studied by the inverse scattering transform technique. In particular, solitons are known to be *stable* under small perturbations and even *superstable*, that is, they are able to survive in collisions.

The nonlinear potential term in (1) arises due to the optic medium’s response to intensive electromagnetic radiation. The “cubic” nonlinearity includes this response only in the first order, and examination of more general nonlinearities is natural. The simplest model which generalized the “cubic” one is the polynomial  $F(i) = i^p + \delta i^q$ ,  $q > p$ ,  $\delta > 0$ , which at the same time poses the features of some models of saturation nonlinearity.

We study the following solitonic properties for the equation (1) (only in 1D for the moment): 1) the stability of solitons and 2) their behavior in collisions. It appears that solitons of the non-integrable NLS equation no longer have the stability properties. In particular, one can expect the phenomena of rebuilding of solitons both due to perturbation and collisions. The rebuilding means that at a long time the solution contains soliton(s) whose parameters are *not* close to the parameters of the initial soliton(s). The details may be found in [1].

The example leads to the conjecture which is formulated below. This conjecture is based on numerous computer experiments which were performed during the IMA experience of the author.

Let solitons be parameterized by the single parameter  $w$ , and let  $N(w)$  denote the  $L_2$ -norm of the soliton  $u_0(x; w)$ . Suppose that  $N(w)$  has a minimum at  $w = w_*$ . (in other words, the system admits the existence of both unstable and stable solitons). Then for  $w < w_*$  there exists some positive  $\lambda_*$  such that the equation (1) has a solution of the form  $U_w(e^{\lambda_* t}, x) = \sum_{k=0}^{\infty} e^{k\lambda_* t} u_k(x; w)$ , where  $u_k(x; w)$  are determined by some recurrent procedure. After a long time, the dominating part of this solution is another soliton with the frequency  $w_\infty > w_*$ . *The conjecture:* if  $u_{t=0} = u_0(x; w) + v(x)$  where  $v(x)$  is small in the appropriate norm, then  $u(x, t) = \overline{U_{\tilde{w}}(e^{\lambda_* t}; x) + o(1)}$  when  $t \rightarrow \infty$ , where  $\tilde{w}$  is close to  $w$ .

## References

- [1] V.E. Grikurov. Soliton’s rebuilding in one-dimensional Schrödinger model with polynomial nonlinearity. IMA Preprint Series # 1320, July, 1995.

Let us consider the following integro-differential equation of hyperbolic type,

$$u_t t - m(\int_{\Omega} |\nabla_x u|^2 dx) \Delta_x u = 0, \quad (1)$$

where  $m$  is a smooth function satisfying  $m(s) \geq v > 0$ , for any  $s \geq 0$ , and  $\Omega$  is any open subset of  $\mathbb{R}^N$ . eq. (??) represents a nonlinear model for the vibrations of strings or membranes and in the last fifty years has been the subject of several studies from the pure mathematical point of view. In spite of the apparent simplicity of Eq.(??) and unlike the well-known results on hyperbolic equations of local type, it is not known whether or not regular solutions develop singularity in finite time. In general not very much is known about the qualitative behaviour of solutions of Eq.(??).

During my permanence at the IMA I took advantage of the workshop on “Singularity and Oscillations” to learn the recent developments of Nonlinear Geometric Optics and to begin a collaboration with Prof. Olivier Guès. The aim of this collaboration has been that of investigating the behaviour of oscillating solutions of Eq.(??) in the case when  $\Omega \equiv \mathbb{R}^N$ . The main result consists in the construction of small amplitude and high-frequency solutions of Eq.(??), i.e. solutions depending on a small parameter  $\varepsilon$ , of the

form

$$\bar{u}(x, t) = \varepsilon^{3/2} \left\{ \sum_{j=0}^{M-1} \varepsilon^j V^j(x, t, \varphi(x, t)/\varepsilon) + O(\varepsilon^M) \right\} \quad (M \geq 1), \quad (2)$$

where the *profiles*  $V^j(x, t, \theta)$ ,  $j = 0, \dots, M-1$ , are smooth functions  $2\pi$ -periodic in  $\theta$  and the *phase*  $\varphi$  is a solution of the characteristic equation of the linearized operator in zero. The presence of the factor  $\varepsilon^{3/2}$  guarantees on one hand the validity of the expansion (??) in a time interval  $[0, T(\varepsilon)]$ , with  $T(\varepsilon) = O(\log(1/\varepsilon))$  and on the other hand that the size of the oscillations is sufficiently large to have a genuine nonlinear perturbation theory. The principal part of the expansion (??) is of particular interest (the so called geometric optics term). In the case of Eq.(??) turns out to be a global (in time) solution of a quasilinear integro-differential transport equation. These results have been recently presented at the congress of the Italian Mathematical Society but the paper is still in preparation.

Solutions of the form (??), depending on a single phase, are called simple wave. Our next goal would be that of studying the Cauchy problem with oscillating initial data, where the interaction resonance) of several waves with different phases renders the problems more difficult and interesting.

### David Sattinger

I found the past year to be extremely helpful in gaining a better perspective on higher dimensional inverse scattering problems; and on the methodology used in industrial problems. My own area of expertise lies in the area of one-dimensional problems and their application to completely integrable systems. The emphasis of the current year was on multi-dimensional problems, and their applications in seismology and inverse conductivity problems.

Here I will describe briefly some of the lectures that made a particular impression on me and that I found useful.

One-dimensional problems tend to be well-posed mathematical problems, both from the standpoint of unique determination of the unknown functions by the data, and from the viewpoint of stable inversion algorithms, such as the Gel'fand-Levitan-Marchenko integral equations, or Riemann-Hilbert problems.

The inversion of multi-dimensional problems, by contrast, generally involves highly unstable algorithms. Some of these algorithms were discussed by Margaret Cheney in her lectures. George Papanicolaou even pointed out that the one-dimensional problems, though well-posed from a theoretical point of view, are nevertheless destabilized by the necessity of selecting only part of the data. (It is impractical to obtain the full set of data for a problem; only partial information is available from the field.)

Recent progress has been made in unique determination of the potentials from the data. This began with the work of Adrian Nachman and continued with Sylvester and Uhlmann. That work has been extended and generalized by Changmei Liu (a student of Nachman's) in her thesis. She reported on her work both in IMA seminars and in PDE seminars in the mathematics department.

Since the full inversion problem is unstable, one must turn to some sort of approximation scheme which aims to recover only rough features of the potentials from singular features of the scattering data. This is the approach being developed by William Symes, who gave a series of lectures on the recovery of singularities of the potentials (such as discontinuities) from the data. Symes' approach is based on the theory of hyperbolic partial differential equations.

Another approach to approximations to inverse problems was discussed in an elegant set of lectures by David Colton of the University of Delaware. Colton discussed the electric far-field operator; this is very nicely developed mathematical theory of the asymptotics of the long range electric field. It gives some information on the scattering object.

Bob Burridge spoke on wave phenomena in nonlinear media and its analysis. In addition, Burridge has developed one-dimensional methods and their application to statefield plane layered geometries. This work displays the interrelationship between certain inverse schemes and explicit finite difference schemes for solving hyperbolic systems, and I have been looking at some of these papers.

The conference in June, organized by Barry Simon, addressed some of the developments in one-dimensional inverse scattering. Fritz Gesztesy and Percy Deift both talked on problems in mathemati-

cal physics coming from integrable systems. Deift is by now the leading researcher in Riemann-Hilbert problems and their applications to inverse problems in mathematical physics.

Finally, I would like to mention discussions with Postdoctorial fellows Sarah Patch and Shih-Hsien Yu on some of their work.

Patch is working on an interesting kind of inverse problem generated by a kind of Markov process, in which photons enter a box, bounce around randomly, and eventually exit. If one knows the relationship between the entering photons and the exiting photons, one would like to recover the internal transition probabilities. So far, the problem is solved only for the simplest system. The problem displays a number of attributes common to inverse scattering problems: over-determination of data, factorization problems, and so forth. Unfortunately, even at the  $2 \times 2$  level, the problem is quite difficult, and it remains to be seen whether the full problem will turn out to be tractable.

Shih-Hsien Yu showed me his work the use of weighted norms for finite difference schemes in conjunction with non-linear hyperbolic conservation laws. I had introduced such weighted norms for the study of stability of traveling waves of nonlinear parabolic systems some years ago, and Yu has shown that similar weighted norms can be used to prove the stability of a discrete shock profile. This appears to be a promising direction.